Combining Vehicle Routing with Forwarding

– Extension of the Vehicle Routing Problem by Different Types of Sub-contraction

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Abstract:

The efficiency of transportation requests fulfillment can be increased through extending the problem of vehicle routing and scheduling by the possibility of subcontracting a part of the requests to external carriers. This problem extension transforms the usual vehicle routing and scheduling problems to the more general integrated operational transportation problems. In this contribution, we analyze the motivation, the chances, the realization, and the challenges of the integrated operational planning and report on experiments for extending the plain Vehicle Routing Problem to a corresponding problem combining vehicle routing and request forwarding by means of different sub-contraction types. The extended problem is formalized as a mixed integer linear programming model and solved by a commercial mathematical programming solver. The computational results show tremendous costs savings even for small problem instances by allowing subcontracting. Additionally, the performed experiments for the operational transportation planning are used for an analysis of the decision on the optimal fleet size for own vehicles and regularly hired vehicles.

Keywords: VRP with subcontracting, own fleet, external carriers, balancing different modes of fulfillment, truck fleet size, cherry-picking

Acknowledgement

This research was supported by the German Research Foundation (DFG) as part of the Collaborative Research Centre 637 "Autonomous Cooperating Logistic Processes – A Paradigm Shift and its Limitations" (Subproject B9).

1. Introduction

Transportation planning requires both, decisions on the available transport resources and decisions on the deployment of the used resources. Therefore, transportation orders (represented by customer transportation requests) must be assigned to resources for fulfillment while the operations or conditions for the usage of each resource have to be determined. Most freight forwarding companies have to cope with a strongly fluctuating demand on the transportation market which varies considerably over time. Aside from these long-term fluctuations they have to manage the daily variations of their volume of orders. Each day a varying number of requests are received from customers on short call. Therefore, freight forwarding companies have to ensure that enough resources will be provided. On the other hand, the fixed costs of the own vehicle fleet (consisting e.g. in the wages for drivers, taxes for vehicles, and amortization costs) force to keep the own fleet small in order to reach a maximal utilization of the fleet. Thus, the number of own vehicles is often reduced since it makes no sense for a forwarder to provide enough transportation capacity able to cover the peaks of a volatile volume of orders. Usually, only a part of the upcoming requests is fulfilled by own transportation resources. All the remaining orders are outsourced. Using own vehicles for the execution of tasks is called self-fulfillment, while the outsourcing of transportation requests to external carriers is called subcontracting. A rigorous reduction of the own fleet size is mostly profitable because it allows the so called cherry-picking which means to perform only the most suitable requests in a very efficient manner by self-fulfillment. But cherry-picking does not really make sense unless the remaining requests are fulfilled in a cost-efficient way as well.

The classical Vehicle Routing Problem (VRP) has first been introduced and investigated by Dantzig and Ramser (1959). Ball *et al.* (1983) have proposed the option for transportation requests fulfillment by using external carriers. Chu (2005) presented a model considering simultaneously the determination of routing a heterogeneous own fleet and the selection of forwarding some requests singly to external carriers. Bolduc *et al.* (2007) have revised some errors in Chu's paper and proposed an advanced heuristic for the problem. Extending the usual planning problems of vehicle routing and scheduling by the additional possibility of subcontracting a part of the requests raises two main questions for the research on operational transportation planning. The first question concerns the long-term planning horizon and refers to the optimal size of the own fleet. The second question affects the short-term planning horizon and applies to the selection of requests to be performed by self-fulfillment and those to be fulfilled by subcontracting.

This selection process cannot be reduced to a simple "either-or" alternative in the sense of an isolated "make-or buy" decision for each single request supported by an adequate and easily applicable comparison method. Instead, the complex decision for self-fulfillment or subcontraction has to take into account dependencies among all available requests since they are to be clustered to common bundles and the resulting transportation costs depend on the bundling performed by the dispatchers of the freight forwarding company. The process of constructing an entire fulfillment plan for self-fulfillment and sub-contraction with the highest reachable quality corresponds to solving the combined vehicle routing and forwarding problem which is also called the integrated operational transportation problem (IOTP). Although this problem is very important for forwarders in practice there exist only few approaches that investigate and solve that problem in literature. A survey of existing approaches can be found in Kopfer and Krajewska (2007).

Caused by the tendency to outsource a tremendous part of the daily transportation requests to external carriers, the need for solving the IOTP in practice is even soaring. The IOTP concerns almost all forwarders with an own fleet of vehicles. For each request to be executed during the next planning period they have to choose an appropriate mode of fulfillment (mode-selection), i.e. they must decide whether a request should be executed using own resources or whether it should be forwarded to an external carrier. In order to minimize the costs of the own fleet, the forwarders have to solve a usual vehicle routing and scheduling problem for all those requests that are dedicated for self-fulfillment. The fulfillment costs incurred by the engagement of carriers can also be influenced for the set of all forwarded requests by means of a skillful operational planning of the employment of subcontractors. This usually can be reached by building favorable bundles of requests which are tied together and are assigned to be forwarded to an elected carrier. The corresponding planning process is called freight consolidation. The goal of the forwarder during the freight consolidation process is to minimize the incurring external freight costs. The solution space of the freight consolidation problem is built by all feasible choices on different possibilities of concentrating requests to bundles and all choices on assigning the constructed bundles to elected carriers of diverse types.

The purpose of this paper is to identify the different modes of sub-contraction and to investigate the interdependency of these modes and self-fulfillment. The objective is to take advantage of incorporating diverse types of sub-contraction in the classical vehicle routing and scheduling problem by minimizing the total fulfillment costs which are composed of both, the variable and fixed costs of the own fleet and the total external carrier costs. In order to get meaningful and illuminating results the investigation concentrates on a type of the IOTP which includes all discussed fulfillment modes, but which is easy enough to be solved to optimality.

This paper is organized as follows. In Section 2 the expansion of the usual problem of vehicle routing and scheduling to the IOTP by incorporating different types of subcontracting is discussed. Approaches developed to solve such problems are reviewed in Section 3. A mathematical model for the expansion of the plain VRP to a basic type of the IOTP is introduced in Section 4. Computational results are presented in Section 5. Finally, conclusions and an outlook for future research are drawn in Section 6.

2. Expansion of Vehicle Routing Problems by Subcontracting

The IOTP is a complex decision problem which consists of several planning levels with different sub-problems shown in Figure 1. The sub-problems of mode-selection, vehicle routing and scheduling, as well as the sub-problem of freight consolidation are strongly dependent on each other. That is why the optimal solution of the entire problem can definitely be reached only by approaches which perform the solution process for all involved sub-problems simultaneously. Good suboptimal solutions can only be generated by heuristics which take the dependencies between the sub-problems into account, for instance by a tabu search algorithm solving the routing problems in the different sub-problems and allowing moves for swaps and insertions which cross the borders of the involved sub-problems. The simultaneous optimization of both sub-problems of the IOTP (i.e. self-fulfillment and sub-contraction) is aspired. Since the IOTP is an extension of the usual vehicle routing the solution space of the IOTP is greater than that of a corresponding VRP and that is the reason for the superiority of the solutions of the IOTP compared to those for plain self-fulfillment.

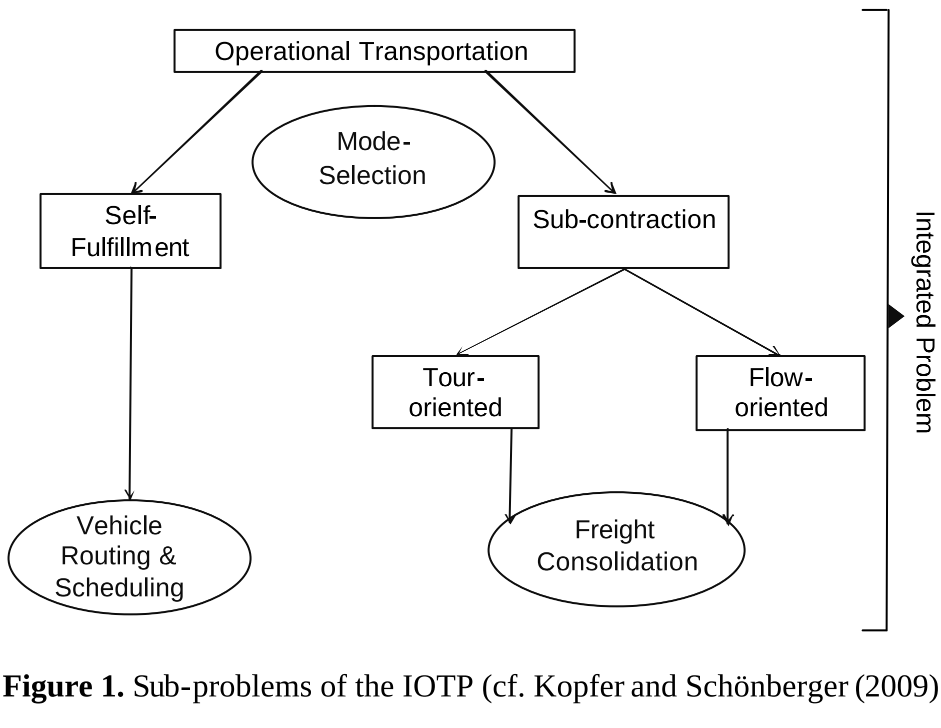

Figure 1. Sub-problems of the IOTP (cf. Kopfer and Schönberger (2009))

Due to the application of complex and diverse methods for freight calculation, the complexity of the IOTP is very high. Figure 1 shows the relations between the involved subproblems. Applying tour-oriented contracts for sub-contraction complete tours are transferred to external carriers and the transportation fees depend on the attributes of the tours planned for the execution of forwarded requests. In contrast, flow oriented contracts are based on the flow of goods caused by transferring bundles of forwarded requests. Most freight forwarding companies use several, alternative forms for compensating external carriers for their employment. Of course, for each single carrier the form of compensation is determined in advance by means of a contract between the forwarder and the carrier. But since we consider the employment of several carriers for the fulfillment of the whole set of upcoming requests, we have to choose between several forms of compensations simultaneously, i.e. we have to assign each request to an appropriate type of sub-contraction with its specific type of freight calculation for the payment of carriers. The type of sub-contraction (type of payment) to be applied depends on the choice of the entrusted carrier, since the contracts with the carriers are fixed.

We consider three types of sub-contraction in this paper. The first two types are tour-oriented and the third one is flow-oriented. In case of a tour-oriented type of sub-contraction less-than-truckload requests are combined to full-truckload orders and the resulting tours are forwarded to external carriers. Applying the first type of sub-contraction, the carrier is entrusted with a complete tour and the payment for the execution of the tour depends on the length of the route to be performed. The calculation of the transportation fee is based on a fixed tariff rate per distance unit, i.e. the amount of payment is calculated by multiplying the length of the entrusted route with the agreed tariff rate. This type of forwarding requests is called sub-contraction on route basis. If sub-contraction on route basis is applied, there are no fixed costs for the forwarding company incurred by the usage of external vehicles. But compared to the usage of own vehicles the variable costs for route based sub-contraction are higher than those for self-fulfillment as the payment of the forwarder has to cover a part of the fixed costs of the carrier.

The second type of sub-contraction results from paying the subcontractors on a daily basis independent of the size of the forwarded tours. In this case an external carrier gets a daily flat-rate and has to fulfill all the received requests of a single day up to agreed distance and time limits. This type is called sub-contraction on daily basis. Costs related to both tour-oriented sub-contraction types (i.e. route based and daily based sub-contraction) as well as the comparison of these types of sub-contraction to the typical costs for self-fulfillment are shown in Figure 2. With respect to the costs the "degree of activity" in Figure 2 can be replaced by the length of the executed tours. Figure 2 simplifies the situation by assuming that the turnover also would linearly depend on the degree of transportation activity, possibly measured by the total length of all executed tours.

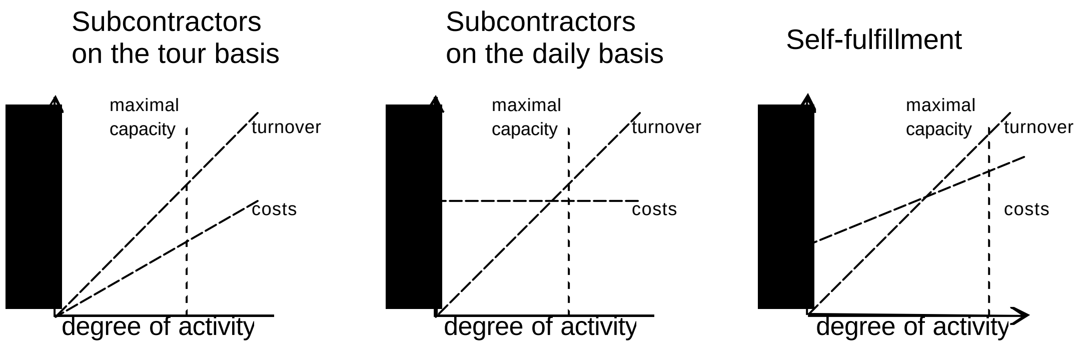

Figure 2. Comparison of costs for different types of sub-contraction and self-fulfillment (cf. Kopfer and Krajewska (2007))

The third type of sub-contraction is characterized by a payment for the pure transportation service performed by the carrier and not on the basis of travelled distances. The transportation service is measured by the extensiveness of the flows representing the transport of the goods of the transferred requests. This type is called sub-contraction on flow basis. The fee due for payment depends on the flow of goods related to the forwarded requests. The transportation flows reach from the source of the goods of the forwarded requests to the destinations of these goods and might be combined to consolidated flows according to the solution of an underlying flow problem (cf. Krajewska and Kopfer (2009)). The amount of payment for flow based sub-contraction arises from the length of the transportation flows and from the amount of goods to be transported on those flows.

An analysis of existing operational transport optimization systems on the software market for freight forwarders has shown that the problem is underestimated. There is no suitable system for freight consolidation on the market, and a system for integrating self-fulfillment and subcontracting is not available, anyway. Due to the lack of software, the problem of splitting the request portfolio into a self-fulfillment and a sub-contraction cluster is solved manually by the dispatchers of the forwarder. An appropriate software support is only available for the sub-problem of self-fulfillment (i.e. for vehicle routing and scheduling). But finding good solutions for the global planning task of splitting the combined problem into sub-problems is even more important than generating high quality solutions for a single sub-problem, since an unfavorable assignment of requests to fulfillment modes may have a more severe impact on the total solution quality than the generation of moderate plans for vehicle routing or freight consolidation.

In practice, planning of the combined routing and forwarding problem is made hierarchically. In the first place the most attractive requests with high contribution margins are planned into the self-fulfillment cluster until all own vehicles are charged to capacity. Here, the schedulers can be supported by software that optimizes the sub-problem of building round routes for a given set of vehicles in the own fleet. Then the other types of sub-contraction are also planned hierarchically. They are considered in a sequential fashion, first planning the forwarding according to the route based sub-contraction type completely, followed by the planning of the daily based sub-contraction type and finally by the flow-based type. Following the above procedure commonly used in practice, the mutual dependencies between the planning for the different fulfillment-modes are ignored and consequently the advantages of simultaneous planning are lost.

3. Approaches for the Integration of the Clusters

The IOTP consists of three sub-problems: splitting the requests into disjoint clusters for different fulfillment modes, cost optimization for the set of requests performed by self-fulfillment (i.e. assignment of requests to vehicles as well as sequencing for vehicle routing and scheduling), and cost optimization (or calculation) for the set of requests dedicated for sub-contraction (with several different types of subcontracting). Different methods of combining these three sub-problems result in different types of the IOTP with different relations between self-fulfillment and sub-contraction. There are three main approaches for combining the involved sub-problems: hierarchical, semi-hierarchical and global integration. These three approaches of integration are shown in Figure 3.

In case of a hierarchical integration (multi-stage planning), the total request portfolio is first split into two subsets that are assigned to the clusters by applying a simple decision rule. This rule does not anticipate the attributes of optimal or near-optimal solutions of the involved subsets. After the splitting into two subsets for self-fulfillment and sub-contraction has been completed, the cost optimization process (possibly cost calculation for the sub-contraction) is performed inside each cluster independently. Such an approach is presented by Chu (2005). When the volume of requests exceeds the available capacity of the own fleet, while time window constraints prevent the extension of the routes, subcontractors have to be involved. Thus, the main idea of the hierarchical planning of Chu (2005) is to choose as many requests as possible for self-fulfillment and to select them in advance on the basis of a costs assessment of tours, and then to optimize the routes for the own fleet. Afterwards, the costs for subcontracting the remaining requests are just calculated, since the freight calculation is performed independently for each request applying a tariff rate for single requests. Due to the level of the tariff rate, sub-contraction is always more expensive than self-fulfillment. The heuristic proposed by Bolduc *et al.* (2007) uses the same general approach as Chu's algorithm while their improvement concentrates on a better solution of the sub-problem to be solved for self-fulfillment.

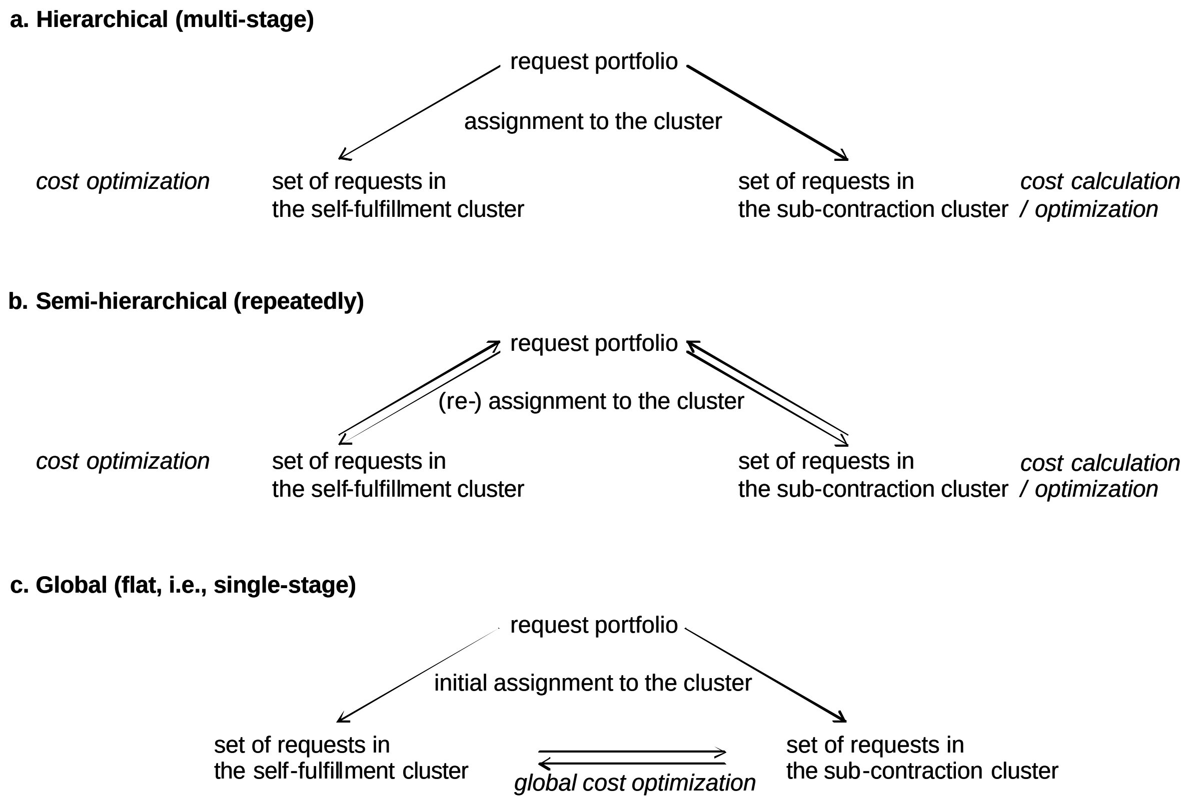

Figure 3. Different types of cluster integration (cf. Kopfer and Krajewska (2007))

The semi-hierarchical approach, e.g. in Pankratz (2002), runs repeatedly by reassigning the requests to clusters in an iterative process. In the first step the solution process builds sets of requests (bundles) which are to be handled in common. Then these bundles are assigned either to the self-fulfillment cluster or to the sub-contraction cluster. Next, different optimization procedures run in the clusters for each bundle separately. They perform the sequencing and scheduling for each bundle in the self-fulfillment cluster and the cost optimization for each bundle in the sub-contraction cluster. Afterwards, using a Genetic Algorithm, new proposals for splitting and bundling are generated by reassigning the requests to the clusters. The bundles of these new solutions are also optimized and evaluated, and so on. As the optimization tasks for the bundles in both clusters cause high time-consumption, the semi-hierarchical planning approach allows changes concerning the division of the request portfolio into the clusters only on the basis of cost estimations and not on the basis of exact optimizations of the sub-problems of the two clusters.

The global (flat) integration, e.g. in the approaches of Schönberger (2005) and Krajewska (2008), is oriented towards the global view at the total costs of self-fulfillment and sub-contraction and tries to minimize these costs holistically. The meta-heuristics used in the presented flat approaches assume that the requests are initially assigned to one of both clusters. Then the cost optimization procedure takes place by altering this initial solution in several iterations. In single iterations of the optimization procedure, the integrated problem is not divided into different sub-problems which are solved by assessment, but there exists a uniform problem representation with a complete implementation plan for all requests. The modification of such a plan for the next iteration runs on the global level. In order to get the next modified plan the requests are shifted not only at other positions within one cluster, but are also possibly shifted from one cluster to a position in another cluster. Consequently, a request can be planned out of the sub-contraction cluster and assigned to a route of an own vehicle, and vice versa.

In particular, Stumpf (1998) as well as Savelsbergh and Sol (1998) can be classified as global planning approaches, as there exists no difference between the strategies for planning the own vehicles and the vehicles of subcontractors in those algorithms; i.e. the same planning procedures are applied for all fulfillment-modes. The optimal routes are aspired for each mode and the requests are shifted between all the routes as well as within one particular route.

Almost all approaches for the IOTP presented in literature concentrate on the extension of a specific type of vehicle routing and scheduling by only one single type of sub-contraction. An advanced approach combining several concurrent types of sub-contraction with vehicle routing and scheduling is presented in Krajewska (2008) and Krajewska and Kopfer (2009). In that approach the PDP-TW is extended by several types of sub-contraction based on the payment for tours and on the payment for flows of goods. The resulting complex decision problem for transportation planning is solved using a tabu search algorithm.

A comparison between the self-fulfillment and sub-contraction mode as well as an investigation of a competing usage of these modes can be found in Schönberger and Kopfer (2009) and in Krajewska (2008). Schönberger and Kopfer (2009) analyze the benefits of the combination of sub-contraction with self-fulfillment in volatile order situations. They use the additional mode of sub-contraction for an enhancement of the flexibility and service quality in case of an overstrained own fleet. Krajewska (2008) describes and solves an IOTP consisting in a global integration of the PDP-TW with the following three types of sub-contraction: route basis, daily basis, and flow basis. She proposes a tabu search algorithm for the solution of that operational planning problem allowing a mixed usage of self-fulfillment and sub-contraction. The proposed tabu search algorithm is also used for experiments on the mid-term planning level by comparing problem instances with different fleet sizes. Since the IOTP has not yet been solved exactly by any optimization algorithm it has not been possible to perform a benchmark for the proposed algorithm. So, it cannot be judged to which extend the results are disturbed by the aberration from the exact solution. The planning situation investigated by Krajewska (2008) includes time windows and is typical for freight forwarding companies in practice. Time windows have a perturbing and complicated effect on the absolute comparison of fulfillment modes since they are treated differently in the various modes with respect to costs and feasibility. Because of their strong and unpredictable influence on the mix of different fulfillment modes time windows are omitted in this paper. This will concentrate the analysis on the investigation of the basic reasons for choosing a fulfillment mode and will keep computational experiments as simple as possible.

None of the existing approaches for integrating self-fulfillment and subcontracting presented in literature really tries to solve the entire IOTP simultaneously for all involved subproblems. This is due to the high complexity of this problem. But an exact optimization of small problem instances may render some important theoretical insights on the relations between different fulfillment modes. In the following section of this paper a totally integrating approach is pursued by using a mixed integer linear programming (MILP) model.

4. A Mathematical Model for Combined Vehicle Routing and Forwarding

In order to allow a straight competition between the considered fulfillment modes, to enable experimental computations with exact solutions, and to investigate the resulting mix of modes in an unbiased situation, the plain VRP is chosen for the combination with the above mentioned types of sub-contraction. The combined problem is modeled as an MILP and is solved to optimality for small test instances.

Given a set of vertices $V = \{0, \ ..., \ n\}$, the VRP is concerned with the optimum routing of a fleet of trucks between the depot ($i = 0$) and a given set of n customers i ($i \in V \setminus \{0\}$) which are to be delivered with goods available at the depot. The distances d_{ij} ($i, j \in V \setminus \{0\}$) between all customer locations (i, j) as well as the distances d_{0j} ($j \in V \setminus \{0\}$) from the depot to each customer are known. Cycles from a customer location to itself are prohibited, i.e. $d_{ii} = +\infty$ ($i \in V$), and all distances are symmetric, i.e. $d_{ij} = d_{ji}$ ($i, j \in V$). The quantity q_i of the demand for goods is given for each customer i. Each vehicle k has a limited capacity Q. Therefore, in general several vehicles are needed for customer satisfaction. The planning task of the VRP is to find an assignment of customers to vehicles and to find for each vehicle a sequence of serving its customers in such a way that all customer demands are satisfied and the total mileage travelled by the fleet is a minimum, while the restrictions of the capacity limitation of the vehicles are met.

Now, the extension of vehicle routing to a combined IOTP will be demonstrated and investigated for the VRP, i.e. the VRP is extended to the Vehicle Routing and Forwarding Problem (VRFP). Self-fulfillment and three sub-contraction types are used for the execution of requests. For self-fulfillment a homogeneous fleet with a limited number of vehicles is available. The own fleet is represented by a set K_s holding m_s equal vehicles. A cost rate cd_s per travel unit is used to calculate the variable costs for the self-fulfillment of a tour. The maximal tour length of any own vehicle is limited by $d_{\max s}$. Additionally, each own vehicle is associated with fixed costs defined as cf_s. Of course, these fixed costs cf_s do not affect the optimal solution resulting from the operational planning but they are of importance for the long-term analysis of the overall cost structures for request execution.

Long-term agreements establish the conditions and the amount of payment for the employment of subcontractors. The employed vehicles of external carriers are equal to the forwarder's vehicles with respect to type and capacity. Moreover, it is assumed that within one type of sub-contraction all carriers are equal with respect to the applied tariff. On the basis of tour-oriented contracts, vehicles can be hired from subcontractors for an exclusive use by the forwarder. Not all of the available vehicles owned by a subcontractor have to be in service. Thus, a payment is made only for those external vehicles that are actually used. On the basis of flow-oriented contracts, the costs for forwarding requests depend on the length and amount of transportation. Usually, the requests forwarded to a carrier by flow based sub-contraction are less-than-truckload transportation orders. The carrier will try to combine the received requests together with further requests from other shippers to full-truckload bundles. This planning process of the carrier is not visible to the forwarder.

For the first type of forwarding (route basis) the set K_r consisting of m_r vehicles of subcontractors paid on route basis is disposable. The tariff rate cd_r per travel unit of a vehicle $k \in K_r$ corresponds to the cost rate cd_s for the own fleet, but it is higher than cd_s, i.e. $cd_r > cd_s$. The tour length of any vehicle from K_r is not allowed to exceed the limit $d_{\max r}$.

For the second type of forwarding (daily basis) a set K_d of m_d vehicles is available. These vehicles can be hired from subcontractors paid on daily basis. Only a flat-rate cf_d per day has to be paid for any actual used vehicle of K_d. The maximal tour length of any vehicle from K_d is limited by $d_{\max d}$.

The third type of forwarding is realized by sub-contraction on flow basis. The presented model does not take into account a freight consolidation by means of flow optimization, i.e. all requests in the flow cluster are forwarded separately. The payment C_f depends on the length and the volume of transportation. The length of transportation is given by the distance d_{0i} from the depot to the customer location of request i. The transportation costs are assumed to depend on the transportation length with a linear cost rate cd_f per distance unit from the depot to the customer i. The volume of transportation is given by the demand q_i. With respect to the amount of transportation the sub-contraction costs are calculated on a pro-rata basis. The pro-rate function $p(q_i)$ depends on q_i and reflects the degree of utilization of a vehicle. In order to get a simple MILP the pro-rata function $p(q_i)$ is assumed to be linear or piecewise linear, e.g. $p_1(q_i) = \frac{q_i}{Q}$, $p_2(q_i) = 1$ or $p_3(q_i) = \max\{p_1, \quad a \cdot p_2\}$ with a suitable parameter a. Altogether, with respect to the length and amount of transportation the forwarding costs are computed to $C_f = p(q_i) \cdot d_{0i} \cdot cd_f$.

The solution of the VRFP consists in a feasible total fulfillment plan with the minimal execution costs. The objective function C comprehends the entire costs including the costs C_s for self-fulfillment, the costs C_r for sub-contraction on route basis, the costs C_d for sub-contraction on daily basis, and the costs C_f for sub-contraction on flow basis:

$$\min C = C_s + C_r + C_d + C_f \tag{1}$$

Altogether there is a set $K = K_s \cup K_r \cup K_d$ with m vehicles ($m = m_s + m_r + m_d$) that can be used for building tours. Let x_{ij}^k be a binary variable such that $x_{ij}^k = 1$ if and only if any vehicle $k \in K$ travels between customer locations i and j. For the formulation of the constraints of the VRFP we need two additional binary variables y_i^k and z_i. The binary variable y_i^k denotes the assignment of customers to vehicles, i.e. $y_i^k = 1$ if customer i is served by vehicle k. Let z_i be the binary variable such that $z_i = 1$ if and only if the request of customer i is assigned to be fulfilled by flow based sub-contraction. The components of the fulfillment costs C_s, C_r, C_d and C_f can be calculated according to the equations (1a), (1b), (1c) and (1d).

$$C_s = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K_s} x_{ij}^k \cdot d_{ij} \cdot cd_s + m_s \cdot cf_s \tag{1a}$$

$$C_r = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K_r} x_{ij}^k \cdot d_{ij} \cdot cd_r \tag{1b}$$

$$C_d = \sum_{i \in V \setminus \{0\}} \sum_{k \in K_d} x_{0i}^k \cdot cf_d \tag{1c}$$

$$C_f = \sum_{i \in V \setminus \{0\}} z_i \cdot p(q_i) \cdot d_{0i} \cdot cd_f \tag{1d}$$

The feasibility of the entire fulfillment plan is assured if each request is assigned to exactly one fulfillment mode and if the constraints for each fulfillment mode are maintained. For aspects of constraint analysis, all vehicles $k \in K$ can be considered together, as only the objective function differs, while all restrictions are alike. Since the sets K_s, K_r, K_d are disjoint, round routes have to be constructed for all own and external vehicles k in a similar way like in a usual VRP with a homogeneous fleet. The difference between the vehicles out of different pools is realized by the components of the objective function summing up the costs for each type of sub-contraction including all vehicles which belong to this type.

Equation (2) assures that each customer is either assigned to exactly one vehicle $k \in K$ or otherwise that the customer i is served by means of flow based sub-contraction. Since $|K_s| = m_s$, $|K_r| = m_r$ and $|K_d| = m_d$, the numbers of actually used vehicles for self-fulfillment, for route based sub-contraction, and for daily based sub-contraction cannot exceed the number of available vehicles of each type respectively.

$$\sum_{k \in K} y_i^k + z_i = 1 \qquad \forall i \in V \setminus \{0\} \tag{2}$$

In equation (3) it is assured that each customer assigned to a vehicle is approached by that vehicle exactly once and that it is left by the same vehicle once. Additionally, equation (3) guarantees that a customer is only served by that vehicle that he is assigned to and that customers served by flow based sub-contraction are not visited by any vehicle at all.

$$\sum_{j \in V} x_{ij}^k = \sum_{j \in V} x_{ji}^k = y_i^k \qquad \forall i \in V, k \in K \tag{3}$$

Equation (4) and (5) enforce that the sum of the demands of all customers served by a vehicle k does not exceed the capacity limit Q of the vehicles. Additionally (4) and (5) prohibit for each tour of a vehicle the execution of short cycles, i. e. they prevent cycles without visiting the depot. So, (4) and (5) guarantee that each vehicle performs a Hamiltonean cycle without exceeding the capacity limit Q.

$$u_i^k - u_j^k + Q \cdot x_{ij}^k \le Q - q_j \qquad \forall i, j \in V \setminus \{0\}, k \in K \tag{4}$$

$$q_i \le u_i^k \le Q \qquad \forall i \in V \setminus \{0\}, k \in K \tag{5}$$

Constraint (6), (7), and (8) enforce that the limits for the maximal length of tours are observed. Finally, the constraints characterizing x_{ij}^k, y_i^k and z_i as binary variables are represented by (9), (10), and (11).

$$\sum_{i\in V}\sum_{j\in V} x_{ij}^k \cdot d_{ij} \le d_{\max s} \qquad \forall k \in K_s \qquad (6)$$

$$\sum_{i\in V}\sum_{j\in V} x_{ij}^k \cdot d_{ij} \le d_{\max r} \qquad \forall k \in K_r \qquad (7)$$

$$\sum_{i\in V}\sum_{j\in V} x_{ij}^k \cdot d_{ij} \le d_{\max d} \qquad \forall k \in K_d \qquad (8)$$

$$x_{ij}^k \in \{0, \quad 1\} \qquad \forall i, j \in V, k \in K \qquad (9)$$

$$y_i^k \in \{0, \quad 1\} \qquad \forall i \in V, k \in K \qquad (10)$$

$$z_i \in \{0, \quad 1\} \qquad \forall i \in V \qquad (11)$$

The objective function (1) together with the constraints (2) to (11) constitute a complete MILP model for the short-term planning of the VRFP. For the strategical fleet size problem, the fixed costs of the own fleet would strongly influence its scale. The best value of the number of own vehicles m_s has then to be regarded as a decision variable and to be determined for the long-term. This can be achieved by substituting the term $m_s \cdot cf_s$ by $\sum_{i\in V\setminus\{0\}}\sum_{k\in K_s} x_{0i}^k \cdot cf_s$.

This substitution yields the new objective function (1') applicable for the determination of the best fleet size, both for the own fleet and for the fleet subcontracted from external carriers.

$$\min \sum_{i\in V}\sum_{j\in V}\sum_{k\in K_s} x_{ij}^k \cdot d_{ij} \cdot cd_s + \sum_{i\in V\setminus\{0\}}\sum_{k\in K_s} x_{0i}^k \cdot cf_s + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K_r} x_{ij}^k \cdot d_{ij} \cdot cd_r$$

$$+ \sum_{i\in V\setminus\{0\}}\sum_{k\in K_d} x_{0i}^k \cdot cf_d + \sum_{i\in V\setminus\{0\}} z_i \cdot p(q_i) \cdot d_{0i} \cdot cd_f \qquad (1')$$

For operational planning the optimal usage of the available fleet is searched while the size (m_s, m_r, m_d) of the own fleet and of the fleet available from subcontractors is restricted according to the decision already made by the forwarder at the strategical level. For long-term planning the values of m_s, m_r, and m_d are set to be great enough so that for all fulfillment modes there will be an oversupply on available vehicles. The objective function (1') then will not only offer the best execution plan, but also the best fleet composition. As the set of constraints allows some k in K not to be designated to any request and the objective function

(1') ensures that these vehicles will not cause any costs, the actual quantity of used vehicles in the optimal execution plan can be directly used for the determination of m_s, m_r, and m_d.

5. Computational Experiments

In this Section some computational experiments for balancing different fulfillment modes at the short-term and at the long-term levels are performed. The commercial mathematical programming solver ILOG CPLEX 11 was used to find optimal solutions of all the test problems, which are generated from the real situation of a forwarder in Central Germany. The comparison between VRP and VRFP indicates how much cost could be saved by incorporating external carriers and to which extend cherry-picking could improve the utilization of the own fleet.

For the realization of computational experiments, five test instances simulating the customers' demands in one week are generated (D1, D2, D3, D4 and D5). There are altogether 20 geographically scattered customer locations. From these 20 customers, a random integer number n ($n = 9,...,12$) of randomly chosen customers are selected for each instance. Demands of chosen customers are then generated according to the Poisson distribution $q_i \sim Poisson(8)$. Details of these test problems can be found in the appendix. Distances between two vertices are rounded to the nearest smaller integers in our computation. Values of the parameters are chosen as follows. It is assumed that the forwarder has four vehicles available for the execution of all these transportation requests. Two of them are own vehicles ($m_s = 2$), one vehicle is paid on route basis ($m_r = 1$) and one vehicle is paid on daily basis ($m_d = 1$). The cost rate per distance unit for an own vehicle is set to $cd_s = 0.8$ monetary units. The values of the other cost/tariff parameters are as follows: $cd_r = 1.7$ monetary units per distance unit; $cd_f = 3$ monetary units per distance unit, $p(q_i) = 1$; $cf_s = 500$ monetary units;

$cf_d = 630$ monetary units. The values set for the remaining parameters are $Q = 25$ tons, $d_{\max s} = d_{\max r} = 850$ distance units and $d_{\max d} = 400$ distance units. Figure 4 illustrates the optimal plan of test instance D1.

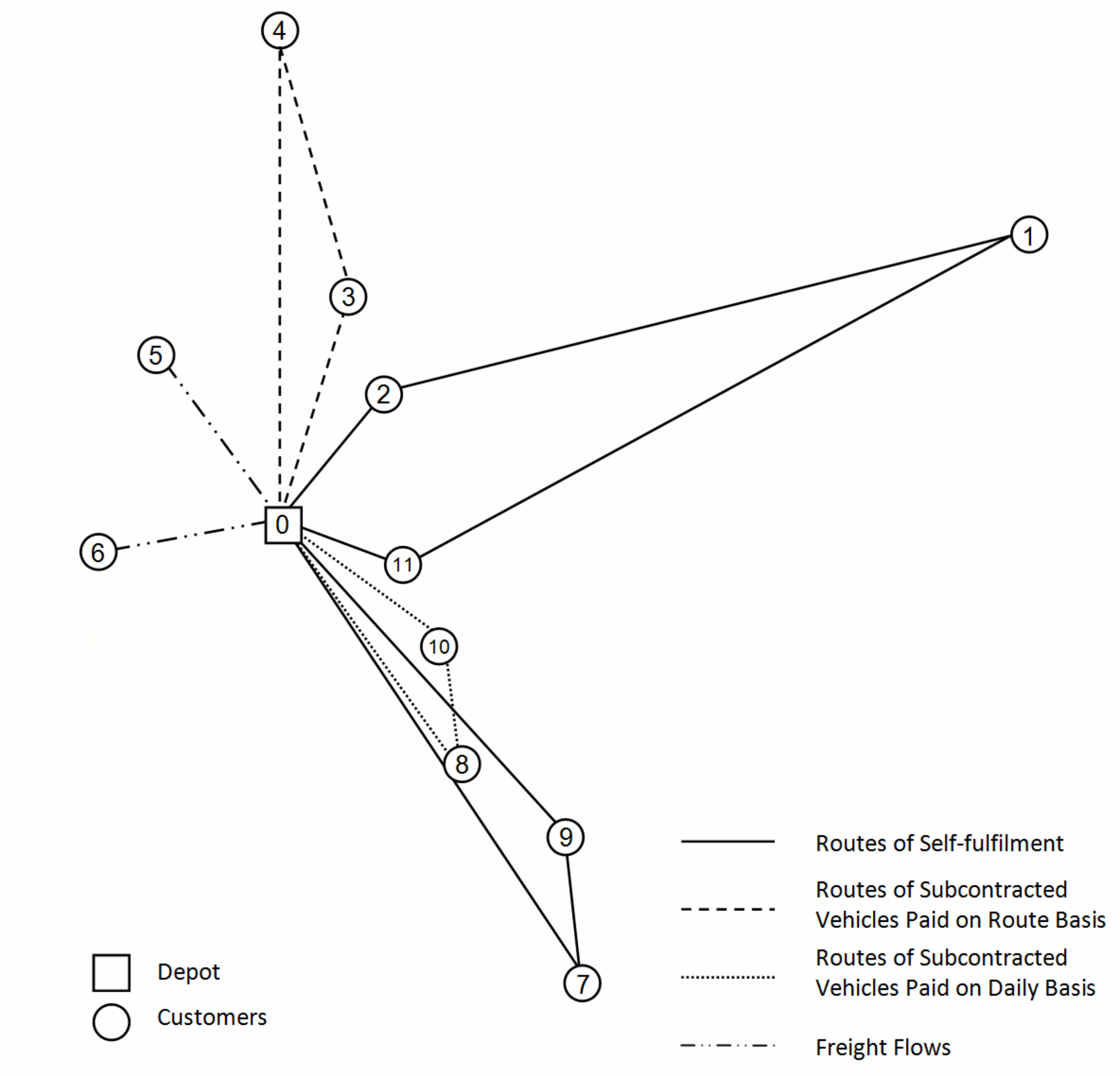

Figure 4. Fulfillment plan for the exemplary Vehicle Routing and Forwarding Problem D1.

In order to explore the potentials of cost saving by sub-contraction, the total fulfillment costs for the VRP and the VRFP are compared. In case of the VRP, the forwarder has to hold an own fleet with at least 5 vehicles if he wants to execute all the requests on his own. In contrast, by integrated planning which is modeled as the VRFP, he can reduce his fleet size incorporating external carriers. Using the same fleet size as shown in Figure 4, i.e. two own vehicles, one vehicle paid on route basis and one vehicle paid on daily basis, the total execution costs for each day (D1, D2, D3, D4 and D5) are presented in Table 1. The total costs for the whole week (sum) are also shown for the variable costs, the fixed costs and the total costs. The weekly total costs for the VRP are regarded as reference and set as 100%. The last row in Table 1 shows the percentage of the arising expenses in relation to that reference value. The utilization factor represents the fraction of the really used own vehicles to the number of available own vehicles for each day. In this simulation, sub-contraction and cherry-picking can reduce the total execution costs for one week by more than 20 percent as well as improve the average utilization of the own fleet from 88% to 100%.

Table 1. Comparison between VRP and VRFP at the operational level

Test Problem	Fulfillment only with own fleet (VRP) $m_s = 5$					Integrated transportation planning (VRFP) $m_s = 2, m_r = 1, m_d = 1$		
	Variable costs	Fixed costs	Total costs	No. of utilized own vehicles	Utilization of the own fleet h (%)	Total costs	No. of utilized own vehicles	Utilization of the own fleet h (%)
D1	2153.6	2500.0	4653.6	5	100	4228.8	2	100
D2	1491.2	2500.0	3991.2	4	80	2847.7	2	100
D3	1704.0	2500.0	4204.0	4	80	3360.6	2	100
D4	1482.4	2500.0	3982.4	5	100	2880.7	2	100
D5	1734.4	2500.0	4234.4	4	80	3325.5	2	100
sum	8565.6	12500.0	21065.6	-	-	16643.3	-	-
%	40.66	59.34	100.00	-	$\bar{h}$: 88	79.01	-	$\bar{h}$: 100

To hold an own fleet increases the self-sufficiency of the forwarder. His own fleet is primarily used for cherry-picking. The carriers paid on route basis are premium subcontractors which are compensated for their service on a relative high level. Usually, they are employed by the forwarder every day. They ensure a reliable execution of the requests assigned to them and enable the forwarder to practice the cherry-picking strategy for the own fleet. The forwarder tries to supply the carriers engaged on route basis with well-bundled tours because he pays them on the basis of the tour length. Compared to his own fleet he will try to assign relatively short tours to route based sub-contraction as the tariff rate for each travel unit is higher than in the case of self-fulfillment.

The carriers paid on daily basis are mainly used to manage the daily fluctuations in the volume of orders and they are indispensable for covering the load peaks. The related costs are relatively high compared to the variable costs of self-fulfillment. Therefore, the break-even point cannot be reached unless the daily limit is actually utilized. In practice, this sub-contraction type is used for requests which do not match very well but are combined to a tour which exploits the maximal tour length for daily paid vehicles.

The flow based sub-contraction is mostly applied for less-than truckload orders forwarded to independent carriers. From the forwarder's point of view it is favorable to entrust these carriers with requests which cannot efficiently combined by the forwarder to routes, for instance because they run into directions where no favorable clustering is possible.

For the long-term planning problem at the strategical level, it is an essential issue for the forwarder, how many vehicles in the own fleet should be held (cf. e.g. Ball *et al.*) and how many vehicles from external carriers should be obtained by signing different types of contracts with subcontractors. We investigate that strategical planning problem for the scenario presented in this paper by comparing the solutions of the VRFP at the short-term level with that at the long-term level. In our experiments, we assume that the number of own vehicles could have been changed everyday of the week and we determine the optimal number of vehicles for each test problem (D1 to D5). The results of this kind of experiments yield a lower bound for the optimal solution for the fleet size at the strategical level, because the solution space of the stragetical planning is enlarged by allowing to vary the number of own vehicles in our experiments instead of an optimal but fixed number of vehicles in the original long-term planning.

For our experiments determining the optimal fleet size for each day anew, we can use the objective function (1') presented in Section 4 by giving m_s, m_r, and m_d values great enough so that they will not have an impact on the restriction of the solution space. However, because of the enormous computational expense, we have obtained the optimal solution for only one instance (D2) by solving the complex problem with CPLEX. Thus, we made experiments for different scenarios of fleet size imposing some more restrictions:

$$\sum_{k \in K_s} y_0^k = m_s \tag{12a}$$

$$\sum_{k \in K_r} y_0^k = m_r \tag{12b}$$

These two constraints imply that for a given scenario, all the available vehicles in the own fleet and those subcontracted on route basis have to execute some of the requests. We then compared the optimal plans of different scenarios with different possible sets of parameter values and found the best scenario for the long-term. The results are shown in Table 2. The total costs for the whole week are also compared with both, the VRP and VRFP test cases. The assumption for this comparison is that the forwarder could have for each day the optimal fleet size. Even under this unrealistic assumption, it is obvious from Table 2 that little costs could be further reduced from the situation with the fleet size m_s = 2, m_s = 1 and m_s =1, which could be seen as a very reasonable solution for the long-term operation.

Table 2. Comparison between operational planning and strategical planning

Test Problem	Operational planning			Lower bound for strategical planning	
	Fulfillment only with own fleet (VRP) $m_s = 5$	Integrated transportation planning (VRFP) $m_s = 2, m_r = 1, m_d = 1$			
	Total costs	Total costs	No. of utilized vehicles: $(m_s / m_r / m_d)$	Total costs	No. of utilized vehicles: $(m_s / m_r / m_d)$
D1	4653.6	4228.8	2/1/1	4167.2	2/2/1
D2	3991.2	2847.7	2/1/0	2847.7	2/1/0
D3	4204.0	3360.6	2/1/1	3360.6	2/1/1
D4	3982.4	2880.7	2/1/0	2848.7	1/2/0
D5	4234.4	3325.5	2/1/0	3325.5	2/1/0
sum	21065.6	16643.3	-	16549.7	-
%	100.00	79.01	-	78.56	-

On the other hand, this simulation assumes deterministic data sets for each single day, usually not available for the strategical decision-making. For the long-term planning, some aggregated information should be gathered. The objective function (1’) can then facilitate the decision-making for the determination of the best fleet size.

6. Conclusions and Future Research

We have analyzed an extension of the VRP to the VRFP allowing the combination of self-fulfillment and sub-contraction of different types. This extension is inline with an often practiced strategy of freight forwarders. By applying optional sub-contraction the total execution costs for transportation can be reduced. This is not surprising, since the solution space of the VRP is enlarged. But the amount of savings that can be achieved in our experiments is impressive.

Future research has to take into account important realistic assumptions of the combined vehicle routing and forwarding. The cost function for sub-contraction on flow basis often has a nonlinear, degressive shape in practical applications. Additionally, flows are merged to jointed flows, containing different combined requests. Thus, the merging of flows yields a minimum cost flow problem to be combined with the routing problem. For further investigations on the effect of combining vehicle routing and forwarding it is necessary to be able to solve medium-sized and large problems of that type. Of course, this can only be achieved by heuristic approaches. Thus, the development of powerful heuristics is an important challenge for the next steps in the research on the IOTP.

Appendix

Details of problem D1

No. i	0	1	2	3	4	5	6	7	8	9	10	11
x_i	0	365	53	35	10	-38	-88	246	136	219	116	52
y_i	0	198	55	125	252	72	-7	-233	-120	-156	-35	-6
q_i	0	9	7	7	14	10	13	14	13	9	8	8

Total demands: 112

Min. number of vehicles needed for the VRP: 5

Details of problem D2

No. i	0	1	2	3	4	5	6	7	8	9	10
x_i	0	90	104	51	10	-75	46	258	219	116	52
y_i	0	28	50	247	252	10	-87	-302	-156	-35	-6
q_i	0	8	8	4	8	9	3	10	8	11	3

Total demands: 72

Min. number of vehicles needed for the VRP: 3

Details of problem D3

No. i	0	1	2	3	4	5	6	7	8	9	10	11	12
x_i	0	90	104	100	35	51	10	-38	-81	46	219	116	52
y_i	0	28	50	147	125	247	252	72	-232	-87	-156	-35	-6
q_i	0	8	9	12	11	7	10	6	5	7	7	9	4

Total demands:95

Min. number of vehicles needed for the VRP: 4

Details of problem D4

No. i	0	1	2	3	4	5	6	7	8	9	10	11
x_i	0	90	104	53	-4	-75	-88	46	258	219	116	52
y_i	0	28	50	55	155	10	-7	-87	-302	-156	-35	-6
q_i	0	7	3	10	7	12	14	10	10	3	7	11

Total demands: 94

Min. number of vehicles needed for the VRP: 4

Details of problem D5

No. i	0	1	2	3	4	5	6	7	8	9	10
x_i	0	53	-4	-75	-81	46	258	136	219	116	52
y_i	0	55	155	10	-232	-87	-302	-120	-156	-35	-6
q_i	0	5	4	16	10	9	8	10	11	7	4

Total demands: 84

Min. number of vehicles needed for the VRP: 4

For customer i, (x_i, y_i) are the coordinates of his location and q_i represents his demand.

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