# A Case Study for a Location-Routing Problem

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## 1 Problem Analysis

In contrast to the classical location-allocation models, the location-routing problem (LRP) integrates vehicle routing and scheduling aspects in the location planning process. A survey and classification scheme of location-routing is e.g. proposed by [3] and [2]. This paper presents a LRP of a company which distributes finished cars. This LRP is decomposed into two steps. In the first step a set of candidate depots is determined by minimizing the sum of the radial distances between depots and customers. Based on the locations found in the first step the vehicle routing problem is used to select the depot with the most attractive location in the second step. This paper is organized as follows. In the following the problem is discussed followed by a solution approach in Section 2. The computational results are presented in Section 3. Finally, the main conclusions of the study are drawn in Section 4.

The core business of the considered company is the transportation of finished vehicles from automotive factories to car dealers in different regions of Germany. The company's distribution network is structured as a Hub&Spoke network with each hub serving the customers within a defined region. This paper focuses on the interregional transportation of cars which will be necessary if a car dealer from one region demands cars of the type which are produces by an automotive factory in the second region. In the current situation interregional transports have to pass both terminals (see Fig. 1(1)). Hence, this interregional transportation proceeding goes along with unnecessary expense, especially if there exists an unbalanced demand between the observed regions. Due to this imbalance the capacity of trucks returning from the one region to the other region is lower at large and leads to a low degree of capacity utilization. Furthermore this proceeding goes along with negative effects for the truck drivers since they are not able to return to their initial point on the

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same day. Thus, a great drawback of the current distribution system constitutes the required tours between the terminals.

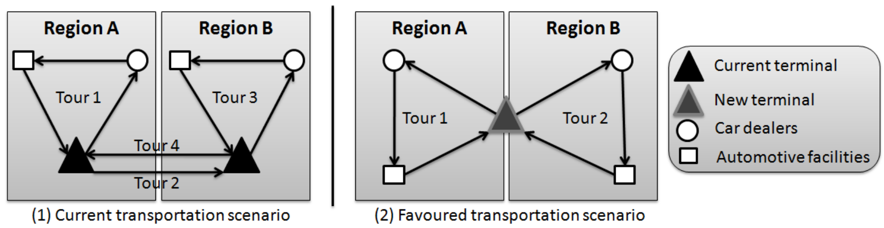


**Fig. 1** The current and favoured interregional transportation scenarios

An additional terminal linking both regions can be used for interregional transportation in order to establish a more efficient scenario (see Fig. 1(2)). Thereby tours between the current terminals will be needless. Cars from automotive facilities in one region will be carried to car dealers in the second region in only two tours via the new terminal. The current terminals then solely are used for the transportation within their region. A demand imbalance between the regions is of little significance since trucks do not have to return to their home depot as in the current interregional transportation scenario. The optimization problem considered in this paper is the LRP for the additional interregional terminal.

## 2 A Sequential Location-Routing Method

The solution process for the LRP is divided into the following two steps:

1. In the first step potential depot locations are determined by means of the Fermat-Weber problem on the one hand and by considering the traffic infrastructure on the other hand. The Fermat-Weber problem is based on a Cartesian coordinate-system and can be solved by Miehle's algorithm ([1]). The attained solution minimizes the Euclidean distances to m fixed customers with distances weighted by the customers' supply and demand amount. Since this does not consider the traffic infrastructure the highways around the attained location should be taken into account, too. Thus, all cities nearby and those close to the highways which connect the considered regions constitute possible candidate locations for the new additional terminal. A diversified cross selection of $q \geq 3$ cities should give a suitable initial network for the second step.

2. In the second step the possible depot locations are rated on the basis of vehicle routing and scheduling results. Hence, implementing a realistic virtual dis-

tribution scenario for each location and choosing the candidate with the lowest expected operational transportation costs will lead to the new terminal location.

The company's vehicle routing problem can be described by means of the vehicle routing problem with backhauls (VRPB) and the split-delivery vehicle routing problem (SDVRP). On each route, all deliveries must be completed before any pickups are allowed. Each customer can be visited more than once since the demand and supply amount of a customer may exceed the vehicle capacity. Since the route length is limited by means of the driving time restrictions the distances between the vertices are described by driving minutes. From the depot 0 all line-haul customers $L = \{1,\ldots,n\}$ with a certain car demand $d$ and all backhaul customers $B = \{n+1,\ldots,n+m\}$ with a certain car supply $f$ should be served. Thus, the problem can be defined over an undirected graph $G = (V,E)$ with vertex set $V = \{0\} \cup L \cup B$ and edge set $E = \{(i,j), i,j \in V\}$. An unlimited number of vehicles each with a capacity $Q$ is available. Each vehicle must start and end its route at the depot. $x_{ijk}$ denotes a Boolean variable equal to 1 if vehicle $k$ travels directly from $i$ to $j$ and equal to 0 otherwise. Moreover $y_{ik}$ defines the quantity of the demand/supply of customer $i$ served by vehicle $k$. The objective is to minimize the total time distance $t_{ij}$ to all customers in both regions.

$$\min z = \sum_{i=0}^{n+m} \sum_{j=0}^{n+m} \sum_{k=1}^{K} t_{ij} x_{ijk} \tag{1}$$

$$\sum_{j=0}^{n+m} x_{0jk} = \sum_{i=0}^{n+m} x_{i0k} = 1, \qquad k = 1,...,K \tag{2}$$

$$\sum_{i=1}^{n} y_{ik} \leq Q, \qquad k = 1,...,K \tag{3}$$

$$\sum_{i=n+1}^{n+m} y_{ik} \leq Q, \qquad k = 1,...,K \tag{4}$$

$$\sum_{k=1}^{K} y_{ik} = d_i, \qquad i = 1,...,n \tag{5}$$

$$\sum_{k=1}^{K} y_{ik} = f_i, \qquad i = n+1,...,n+m \tag{6}$$

$$y_{ik} \leq d_i \sum_{j=0}^{n+m} x_{ijk}, \qquad i = 1,...,n; k = 1,...,K \tag{7}$$

$$y_{ik} \leq f_i \sum_{j=0}^{n+m} x_{ijk}, \qquad i = n+1,...,n+m; k = 1,...,K \tag{8}$$

$$\sum_{i=n+1}^{n+m} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} = 0 \tag{9}$$

$$\sum_{i=0}^{n+m}\sum_{j=0}^{n+m} t_{ij}x_{ijk} \leq T, \qquad k=1,...,K \tag{10}$$

$$\sum_{i=0}^{n+m} x_{ilk} - \sum_{j=0}^{n+m} x_{ljk} = 0, \qquad l=0,1,...,n+m; k=1,...,K \tag{11}$$

$$u_{ik} - u_{jk} + 2*Q \times x_{ijk} \leq 2*Q-1, \qquad i,j=1,...,n+m; k=1,...,K \tag{12}$$

$$1 \leq u_{ik} \leq 2*Q \qquad i=1,...,n+m; k=1,...,K \tag{13}$$

$$x_{ijk} = \{0,1\}, \qquad i,j=0,1,...,n+m; k=1,...,K \tag{14}$$

$$y_{ik} \geq 0, \qquad i=1,...,n+m \tag{15}$$

$$d_i \geq 0, \qquad i=1,...,n \tag{16}$$

$$f_i \geq 0, \qquad i=n+1,...,n+m \tag{17}$$

Constraints (2) impose that every tour has to begin and end at the depot. Constraints (3)-(8) concern the allocation of the demand and supply amount of the customers among the vehicles. While (5) and (6) assure that the entire demand or supply of each vertex is satisfied, (3) and (4) ensure that the quantity delivered or picked up does not exceed the vehicle capacity. Constraints (7) and (8) impose that customer $i$ can only be served by vehicle $k$ if $k$ passes through $i$. (9) guarantees that a truck does not drive to a car dealer after he visited an automotive facility. The abidance of the route length is ensured by restriction (10). Constraints (11)-(13) are the classical routing constraints; constraints (11) impose the continuity of a route while (12) and (13) guarantee that each vehicle performs a Hamiltonian cycle without exceeding the capacity limit of $Q$ cars.

## 3 Computational Results

The regions Southwest Germany (SW) and Southeast Germany (SE) are known to have a significant demand imbalance. In the underlying request data set there were about 50% more cars which had to be moved from SE to SW than vice versa. Therefore, establishing a new terminal between these regions is of great interest for the company. In the first step of the sequential location-routing method we identified four possible candidate locations A-D (Fig. 2).

For the investigation of realistic distribution processes for each possible location we assume the amount of the supply and demand in the regions and construct daily routes to both regions. A truck cannot move more than eight finished vehicles at once. An additional important restriction for the company is defined by the legal driving time regulations. The EC regulations generally allow a truck driver to drive 540 minutes per day. Due to the operating experiences of the analyzed company this route length restriction is reduced further by a buffer of 40 minutes to consider

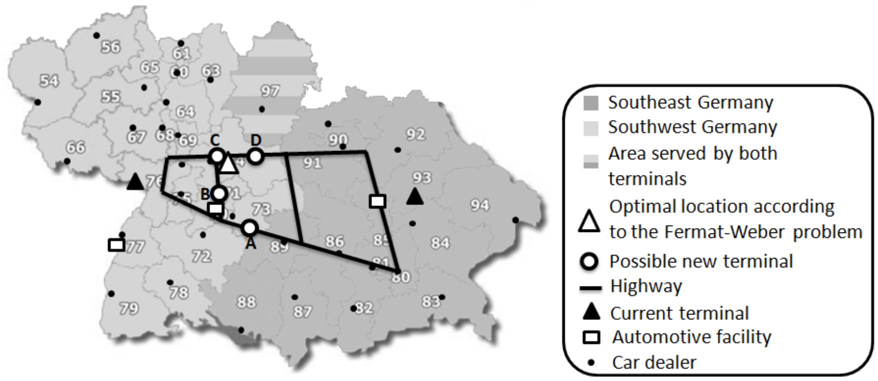


**Fig. 2** Possible depot locations

possible time lags (e.g. traffic jams). The average demand and supply amount in SE and SW is shown in Table 1.

**Table 1** Average amount of demand and supply in Southeast and Southwest Germany

| **Southeast Germany** | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Postcode region | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 97 |
| Demand | - | - | 2 | - | - | 5 | - | 1 | 2 | 2 | - | - | - | 20 | - | 1 |
| Supply | - | - | - | - | - | 3 | - | - | - | - | - | - | - | 63 | - | - |

| **Southwest Germany** | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Postcode region | 54 | 55 | 56 | 60 | 61 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 97 |
| Demand | 2 | - | 3 | - | 3 | 8 | 5 | 4 | 6 | 3 | 2 | 2 | - | 2 | 2 | 3 | 5 | 2 | 3 | 4 | 2 | 2 | 3 |
| Supply | - | - | - | - | - | - | - | - | - | - | - | - | 4 | - | - | - | - | - | 7 | 22 | - | - | - |

Comparing the results of the distribution scenarios in SE and SW yields the rating of the locations. Location C provides the best results and is determined as the potential new terminal. At this point the found new solution with an interregional terminal at location C should be compared with the current situation. Therefore, the current scenario as seen in Fig. 1(1) is analyzed by means of the stated model above. The terminals in both regions serve as the depot for the distribution in SE and SW. Furthermore the connections between these depots have to be added to illustrate the interregional transportation proceeding. Table 2 shows the comparison between the current transportation net and the proposed net with the additional terminal C. Herbey the values are optimal for SE while the solution process for SW has been stopped when an optimality gap of 10% had been reached. For the current route planning the driving time within SE and SW is less than for the route planning with the new terminal since the current terminals are positioned more centrally in the regions. However, like stated in Section 1, the connections between the terminals have to be taken into account. Altogether the current solution leads to 15% additional total driving time compared to the route planning with location C as the new terminal. Furthermore the new solution with an interregional terminal reduces the number of tours by about 28%.

**Table 2** Comparison between the current and favoured distribution proceedings

| Region | Current route planning (Driving time/ Number of tours) | Favoured route planning with terminal C (Driving time/ Number of tours) |
|---|---|---|
| Southeast Germany | 2222/ 5[1] | 6000/ 13[1] |
| Southwest Germany | 3530/ 9[2] | 3863/ 10[2] |
| Connection between the current terminals | 2*2889/2*9 | — |
| Total | 11530/ 32 | 9863/ 23 |

[1] Optimally solved [2] Optimality Gap:10%

## 4 Conclusions

The presented LRP is solved by a method for combining location planning and vehicle routing and scheduling in a sequential way. The case study has shown that the proposed method can lead to significant savings in distribution costs. Providing reliable request data for solving the vehicle routing problem is essential to get a good solution. Nevertheless, reflecting a realistic distribution of goods within the location planning process can provide the opportunity to obtain solutions of higher quality than the classical location-allocation models.

## References

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