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Vehicle and Commodity Flow Synchronization

Jörn Schönberger[1], Herbert Kopfer[1], Bernd-Ludwig Wenning[2], and Henning Rekersbrink[3]

[1] University of Bremen, Chair of Logistics {jsb,kopfer}@uni-bremen.de
[2] University of Bremen, Communication Networks wenn@comnets.uni-bremen.de
[3] University of Bremen, BIBA - Bremer Institut für Produktion und Logistik GmbH rek@biba.uni-bremen.de

1 Introduction

Network freight flow consolidation organizes the commodity flow with special attention to the minimization of the flow costs but the efficiency of transport resources is not addressed. In contrast, vehicle routing targets primarily the maximization of the efficiency of transport resources but the commodity-related preferences are treated as inferior requirements. This article is about the problem of synchronizing simultaneously the use of vehicles, and the flow of commodities in a given transport network. Section 2 introduces the investigated scenario. Section 3 proposes a multi-commodity network flow model for the representation of the flow synchronization problem and Section 4 presents results from numerical experiments.

2 The Flow Synchronization Problem

Related and Previous Work. The synchronization of flows along independently determined paths in a network are investigated under the term *multi-commodity network flow problem* [3]. While most of the contributions aim at minimizing the sum of travel length (or similar objectives) only little work has been published on the assignment of commodity path parts to transport resources with intermediate resource change [1, 2].

An Example of the Synchronization Challenge. Fig. 1 presents an example with three commodities $\gamma \in \{1, 2, 3\}$ in a network $\mathcal{G} = (\mathcal{V}, \mathcal{A}, c, \delta)$ (node set $\mathcal{V}$, arc set A, arc cost function c and arc travel

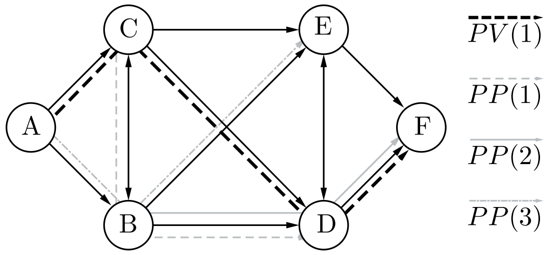

Fig. 1. Example Network with six nodes.

time function δ. The path $PP(\gamma)$ was selected by commodity γ connecting its start node (site of availability of commodity γ) with its target node (site of demand for commodity γ): $PP(1) = (C, B, D)$, $PP(2) = (B, D, F)$ and $PP(3) = (A, B, E)$.

A *package hop* $H^p(i, j)$ describes the movement along the arc $(i, j) \in \mathcal{G}$. The hop $H^p(i, j)$ cannot leave from i towards j before time $a_{H^p(i,j)}$ and $dt_{H^p(i,j)}$ is its actually determined departure time. Transport resources hopping along the arcs in $\mathcal{G}$ execute the hops. A transport resource carries several commodities on a **service hop** $H^S(i, j)$. **Internal service hops** have an unchangeable departure time. In our example, an own vehicle travels along $PV(1) = (A, C, D, F)$. It starts at A at time 0 and visits the subsequent stops C, D, F using three internal service hops $H^S(A, C)$, $H^S(C, D)$ and $H^S(D, F)$. An **external service hop** can be booked at every time. External service hops are provided by logistic service providers (LSP) which are paid according to a previously known tariff for executing hops at the determined time (subcontraction).

A hop sequence $H^p(i_1, i_2), H^p(i_3, i_4), \ldots, H^p(i_{k-3}, i_{k-2}), H^p(i_{k-1}.i_k)$ is concatenated if $i_2 = i_3$, $i_4 = i_5$ and so on. Two concatenated hop sequences $H^1(i_1, i_2), \ldots, H^1(i_{l-1}.i_l)$ and $H^2(j_1, j_2), \ldots, H^2(j_{k-1}.j_k)$ are compatible if and only if $i_1 = j_1$ and $i_l = j_k$. In Fig. 1 the second hop of $PP(2)$ is compatible with the last hop in $PV(1)$ and the concatenated package hop sequence $H^p(C, B), H^p(B, D)$ in $PP(1)$ is compatible with the internal service hop $H^s(C, D)$.

Solving the synchronization problem requires the assignment of each hop sequence associated with $PP(1), \ldots, PP(K)$ to exactly one compatible sequence of (internal and/or external) service hops, so that the following requirements are met: If a package departs from a node then it has been brought to this node before (C1). The hop associated with the first arc in $PV(m)$ ($m \in \{1, \ldots, M\}$) departs at time 0 (C2). The arriving and the departure time of a vehicle at a node coincide and

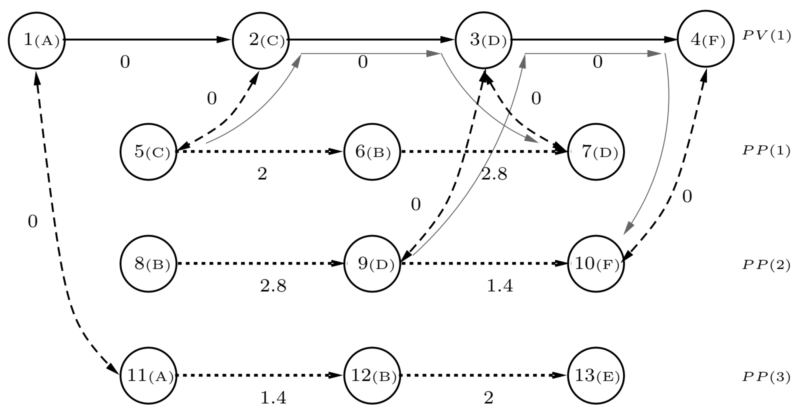

Fig. 2. Matching-Network (in brackets: original labels of the nodes)

neither a loading nor an unloading operation consumes time (C3). The initial hop in $PP(\gamma)$ starts not before time 0 (C4). The difference between the departure times belonging to two consecutively visited nodes i and j is at least $\delta(i,j)$ (C5). If a package hop sequence is assigned to a service hop sequence the first package hop $H^p(i_i,i_2)$ must be available before the first service hop $H^s(j_1,j_2)$ starts, e.g. $a_{H^p(i_i,i_2)} \leq dt_{H^s(j_1,j_2)}$ (C6). The sum of flow costs is minimal (C7).

3 Construction of the Matching Network

The nodes in the vehicle paths $PV(\cdot)$ and in the package paths $PP(\cdot)$ are re-labeled pair wise distinctly by 1, 2, ... In $\sigma(i)$, the original label of node i is stored. Only one departure time must be managed for each node. The re-labeled nodes are shown in Fig. 2. The vehicle path $PV(1) := (A,C,D,F)$ is now $(1,2,3,4)$, the package path $PP(1) := (C,B,D)$ is $(5,6,7)$ and so on.

The set $\mathcal{N}^{veh}$ contains all re-labeled nodes from $PV(1),\ldots,PV(M)$, $\mathcal{N}^{pac}$ consists of the nodes appearing in $PP(1),\ldots,PP(K)$ and $\mathcal{N}^* := \mathcal{N}^{veh} \cup \mathcal{N}^{pac}$. All arcs from the vehicle paths form the set $\mathcal{A}^{veh}$ and all arcs forming the package paths are collected in the set $\mathcal{A}^{pac}$. Vehicle paths and the package paths are connected by *transfer arcs*. A transfer arc connects a node i in a vehicle path with a node j in a package path if and only if the two nodes i and j are the same in graph $\mathcal{G}$. The set $\mathcal{A}^{transfer} := \{(i,j) \in \mathcal{N}^{veh} \times \mathcal{N}^{pac} \cup \mathcal{N}^{pac} \times \mathcal{N}^{veh} \mid \sigma(i) = \sigma(j)\}$

contains all transfer arcs. The solid arcs in Fig. 2 represent the internal service hops, the dotted arcs give the external service hops and the dashed arcs are the transfer arcs for changing the transport ressource.

The *flow synchronization graph* is defined as $\mathcal{G}^* := (\mathcal{N}^*, \mathcal{A}^*, c^*_f, \delta)$ where δ represents the travel times between the nodes along the arcs in $\mathcal{A}^*$. The cost for transferring an arc depends upon the type of the arc. It is assumed that loading and unloading operations do not produce any costs so that traveling along a transfer arc is free. The cost function c^*_f assigning flow costs to the arcs in $\mathcal{A}^*$ is defined by $c^*_f := 0$ if $(i,j) \in \mathcal{A}^{veh} \cup \mathcal{A}^{trans}$, $c^*_f := f \cdot c(i,j)$ if $(i,j) \in \mathcal{A}^{veh}$. If $f > 0$ then the LSP incorporation is more expensive than the usage of the own vehicles but if $f = 0$ then internal and external service hops cause equal costs.

The cost reducing effect of reassigning package hop sequences from a sequence of external service hops to a sequence of internal service hops is demonstrated by the grey arcs in Fig. 2: If commodity $\gamma = 1$ uses the external service hop sequence $H(5,6), H(6,7)$ then the journey from 5 to 7 causes 2+2.8=4.8 money units. In case that the commodity uses the transfer hop $H(5,2)$, then the internal service hop $H(2,3)$ and finally the transfer hop $H(3,7)$ then no costs are accounted.

Multi Commodity Network Flow Model. Let K denote the number of commodities merged in the set $\mathcal{R}$. With $D(i,\gamma)$, we denote the offer $(D(i,\gamma) > 0)$ of commodity γ at node i and the demand $(D(i,\gamma) < 0)$ respectively. The initial node in the path of vehicle $m \in \{1, \dots, M\}$ is denoted by v^+_m and the initial node in the path of commodity γ is named i^+_γ. $\mathcal{M}$ is a sufficiently large number ("Big M").

In order to code the necessary flow decisions, we introduce three families of decision variables. The earliest starting time of hops originating from node i is saved in the decision variable d_i. If the arc $(i,j) \in \mathcal{A}^*$ is used then binary variable u_{ij} equals 1. The portion of the overall demand of commodity γ that flows along the arc (i,j) is represented by the continuous decision variable $x_{ij\gamma}$.

$$\sum_{i \in \mathcal{N}^*} \sum_{j \in \mathcal{N}^*} \sum_{\gamma=1}^{K} c^*_f(i,j) \cdot x_{ij\gamma} \rightarrow \min \tag{1}$$

$$\sum_{j \in \mathcal{N}^*} x_{ij\gamma} - \sum_{j \in \mathcal{N}^*} x_{ji\gamma} = D(i,\gamma) \; \forall i \in \mathcal{N}^* \; \forall \gamma \in \mathcal{R} \tag{2}$$

$$d_{v^+_m} = 0 \; \forall m \in \{1, \dots, M\} \tag{3}$$

$$d_i + \delta(i,j) = d_j \; \forall (i,j) \in \mathcal{A}^{veh} \tag{4}$$

$$d_{i^+_\gamma} \geq 0 \; \forall \gamma \in \mathcal{R} \tag{5}$$

$$d_i + \delta(i,j) \leq d_j \ \forall (i,j) \in \mathcal{A}^{pac} \cup \mathcal{A}^{trans} \quad (6)$$

$$u_{ij} \geq \sum_{\gamma=1}^{M} x_{ij\gamma} \ \forall i,j \in \mathcal{N}^* \quad (7)$$

$$\mathcal{M} \cdot (u_{ij} - 1) + d_j + \delta(i,j) \leq d_j \ \forall (i,j) \in \mathcal{A}^* \quad (8)$$

$$x_{ij\gamma} \geq 0 \ \forall (i,j) \in \mathcal{A}^*, k \in \mathcal{R}, u_{ij} \in \{0,1\} \ \forall (i,j) \in \mathcal{A}^*, d_i \geq 0 \ \forall i \in \mathcal{N}^* \quad (9)$$

Eq. (1) represents C7, (2) addresses C1, (3) corresponds to C2 and (4) to C3. Similarly, (5) ensures C4 and (6) does the same for C5. Finally, (7) and (8) addresses C6.

Test Cases. A set of 36 artificial test cases has been generated, each consisting of a transport graph $\mathcal{G}$, $\alpha = 1, 2$ or 3 vehicles, $\beta = 1, 2, 3$ or 4 commodities and vehicle as well as path proposals $PV(\cdot)$ and $PP(\cdot)$. In the original network $\mathcal{G} := (\mathcal{N}, \mathcal{A}, c, \delta)$ the first 25 nodes from the Solomon instance R104 (dropping the time windows) form the node set $\mathcal{N}$. A minimal spanning tree connecting the 25 nodes generated by Kruskal's algorithm determines the the arc set $\mathcal{N}$. The Euclidean Distance $\delta(i,j)$ gives the distance matrix and the costs $c(i,j)$ for traversing the arc (i,j) are set to one money unit for each distance unit. For each commodity γ, a demand and an offer location is randomly generated and the shortest path $PP(\gamma)$ in $\mathcal{G}$ is calculated using the Dijkstra-algorithm. The path $PV(\cdot)$ starts at an arbitrarily selected node, it continues on a shortest path to a randomly selected stop in $PP(1)$, follows $PP(1)$ for a randomly generated number of hops. Then, it continues on a shortest path to a node in the $PP(2)$ and so on.

4 Numerical Experiments

Setup of the Experiments. The flow synchronization graph $\mathcal{G}^*$ has been set up for each of the 36 test cases and the model (1)-(8) has been solved once with $f = 0$ (internal and external service hops cause the same costs) and once with $f = 1$ (external service hops are more expensive). The lp_solve Mixed-Integer Linear Program solver is deployed for the derivation of the optimal solution for the model (1)-(8). For each scenario (α, β) we have calculated the averagely observed number $l_f(\alpha, \beta)$ of used external service hops and the increase $l(\alpha, \beta) := \frac{l_1(\alpha,\beta)}{l_0(\alpha,\beta)} - 1$ of this number has been calculated. Similarly, we have derived the increase $v(\alpha, \beta)$ of used internal service hops and the increase $t(\alpha, \beta)$ of used transfer hops.

Table 1. Variation of the number of used hops.

	$\alpha = 1$			$\alpha = 2$			$\alpha = 3$		
β	$l(\alpha,\beta)$	$v(\alpha,\beta)$	$t(\alpha,\beta)$	$l(\alpha,\beta)$	$v(\alpha,\beta)$	$t(\alpha,\beta)$	$l(\alpha,\beta)$	$v(\alpha,\beta)$	$t(\alpha,\beta)$
3	0%	0%	14%	-33%	18%	43%	-63%	20%	13%
4	-25%	25%	33%	–	–	–	-77%	-19%	133%
5	-30%	35%	67%	-64%	50%	113%	–	–	–
6	-46%	26%	138%	–	–	–	–	–	–

– not solved within 15 minutes

Results. The observed values for $l(\alpha,\beta)$, $v(\alpha,\beta)$ and $t(\alpha,\beta)$ are compiled in Table 1. If the number of vehicles or the number of commodities is increased then the number of used external service hops decreases ($l(\alpha,\beta)$ decreases). At the same time, the number of used internal service hops is lifted ($v(\alpha,\beta)$ grows up). The utilization of the transfer arcs also increases ($t(\alpha,\beta)$ increases).

5 Conclusions

We have introduced and modeled the problem of synchronizing paths proposals of transport resources and commodities. A mixed integer linear program is proposed. The experimental results demonstrate the intricacy of the flow synchronization. Even for a small number of involved resources and commodities the identification of the best synchronization decisions is not possible. Future research is dedicated to the development of hybrid algorithms that combine a heuristic search with the linear programming in order to shift the border of solvability to larger numbers of vehicles and commodities.

References

1. Borndörfer R, Grötschel M, Pfetsch ME (2004) Models for Line Planning in Public Transport. ZIB-Report 04-10, Berlin
2. Pickl S, Mues C. (2005) Transshipment and Time-Windows in Vehicle Routing. In: Proceedings of International Symposium of Parallel Architectures, Algorithms and Networks
3. Assad AA (1978) Multicommodity Network Flow – A Survey Networks 8:37–91