## ORIGINAL PAPER

# A distributed routing concept for vehicle routing problems

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**Abstract** Traditional solution concepts for the vehicle routing problem (VRP) are pushed to their limits, when applied on dynamically changing vehicle routing scenarios-which are more close to reality than the static formulation. By contrast, the introduced distributed routing concept is designed to match packages and vehicles and to continuously make route decisions especially within a dynamic environment. In this autonomous control concept, each of these objects makes its own decisions. The developed algorithm was entitled Distributed Logistics Routing Protocol (DLRP). But in spite of the restricted suitability of the traditional VRP concepts for dynamic environments, they are still the benchmark for any VRPsimilar task. Therefore, we first present a description of the developed DLRP. Then an adapted vehicle routing problem is defined, which both sides, static and dynamic concepts, can cope with. Finally, both concepts are compared using a tabu search algorithm as a well working instance of traditional VRP-concepts. For a quantitative comparison, four solutions are given for the same adapted problem: the optimal solution as a lower bound, the DLRP solution, a tabu search solution and a random-like solution as an upper bound.

**Keywords** Vehicle routing problem (VRP) · Autonomous control · Distributed logistics routing protocol (DLRP) · Tabu search · Optimisation · Routing algorithm · Transport logistic

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#### 1 Introduction

One opportunity to handle growing dynamics and complexity of logistics systems is to shift from central planning to decentral, autonomous control strategies. The concept of autonomous control is the research area of the German Collaborative Research Centre (CRC) 637 'Autonomous Cooperating Logistic Processes—A Paradigm Shift and its Limitations'. This CRC develops a new concept for dynamic transport networks, which is designed to match goods and vehicles and to continuously make route decisions within a dynamic transport environment. Here, each object makes its own decisions. It is called Distributed Logistics Routing Protocol (DLRP).

In order to evaluate this new concept, we compare it to the traditional solutions for the vehicle routing problem (VRP), shown in this article.

To describe the different approach of autonomous control to transport problems, the developed DLRP is described at first. In contrast to traditional algorithms for the VRP problem, which do static optimisation, this approach tries to control an ongoing dynamic transport process.

Therefore the problem definitions of both sides are different in principal. One basic point is that the VRP is a static problem, because all customers are known at the beginning. In contrast, the problem for the DLRP is a dynamic network formulation: the customers appear continuously and are not known from the beginning. In order to make the results for both sides comparable, an adapted VRP scenario is described which can be handled by the DLRP and traditional VRP algorithms.

Several versions of this adapted VRP scenario were solved by the DLRP on the one hand and by a tabu search algorithm on the other hand. The tabu search algorithm was



taken as a typical traditional VRP algorithm which can manage big solution spaces (see [1, p. 275]).

To rate the results and to evaluate the new concept in a more objective way, four solutions are given for the same adapted problem. In addition to the DLRP and the tabu search solution, lower and upper bounds for the overall vehicle distance were calculated. The optimal vehicle ways are given as best and some kind of random vehicle ways as worst case values. Therefore the new concept is evaluated relatively to a traditional algorithm and secondary to an absolute scale.

# 2 Distributed Logistics Routing Protocol

Real life scenarios of transport processes require a kind of continuous control of logistic objects. Objects like packages and vehicles appear and disappear continuously—the scenario is a dynamic one. After a close consideration on static routing problems like TSP, TRP, VRP, or PDP, to name a few, the need of research on dynamic problems had been stressed in literature recently (see [2–5] or [6] for examples). In the majority of cases, *dynamic* means that not all customer orders are known in advance, in contrast to the traditional static scenario.

The second crucial property of real life scenarios is their size. Our global viewpoint and vision is the control of nearly all transports for example in Germany—to show our long range perspective. Under these circumstances, it is not possible to calculate any optimum—it is not even possible to receive all relevant information for one point of time. All approaches mentioned above follow a central strategy, which has strong restrictions to the scenario size (the number of orders in the mentioned scenarios vary between 100 and 1,000).

Against this background, an autonomous control concept for transport nets was developed. The concept was initially

inspired by internet routing protocols, which are able to find routes through a permanently changing and unknown net. In addition, these concepts are able to deal with very large nets without a central perspective. The basic concept for one data package wanting to get to its destination is the RouteRequest/RouteReply mechanism. This package sends a RouteRequest to all its neighbour vertices, which for themselves sent it ahead to their neighbours. If one vertex notices that it is the destination for this RouteRequest, it sends back a RouteReply to the asking package (for a more detailed description, see, e.g. [7]).

One part of the DLRP is based on this concept. On the basis of Fig. 1 the fundamental procedure of the developed protocol can be illustrated: When a package makes a route decision, it first disannounces its old route (see Fig. 1: RouteDisAnnouncement) and announces its actual planned routes to the vertices involved (see Fig. 1: RouteAnnouncement). An individual vertex thus has information about when how many packages with what destinations will be at its position. Additional information such as restrictions concerning the transport of the packages (e.g. cooling freight) is stored likewise.

If a vehicle needs a route, it sends a RouteRequest to the net—the RouteRequest/RouteReply mechanisms are the same as described above. After receiving several RouteReplies, which are route suggestions with appropriate additional information, a vehicle decides on a route—for example the route with the maximum expected utilisation. This Route is then announced to the involved vertices (see Fig. 1: RouteAnnouncement). This leads to a continuous cooperative structure. The objects in a transport net do not plan their route at the same time. Packages emerge continuously or reach their destination, vehicles replan their routes and so on. At each time there is enough information for any route decision.

The whole DLRP concept offers outstanding advantages for real life applications such as: self-adaptation, manual

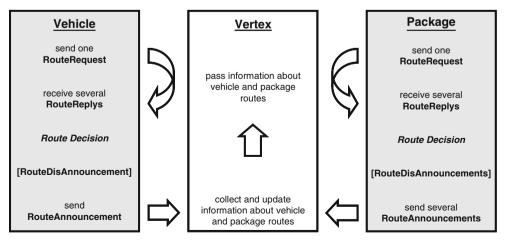


Fig. 1 Scheme of the distributed logistics routing protocol, DLRP (from [8])



intervention, estimation of future net conditions, implicit uncertain knowledge, arbitrary decision processes and arbitrary kind and quantity of information (see [8]). For the solution of the described adapted vehicle routing problems (see next section), the concept was simplified to exclusively optimise the overall vehicle distance. But for all that, the algorithm is still very complex and has many points, directly affecting performance; e.g. which packages are loaded into a vehicle at the vertex: all with the same direction, all with their destination vertex on the vehicle route or other packages. Hence, a detailed description is not possible here. For a more detailed description refer to [8–10].

## 3 Adapted vehicle routing problem

Unfortunately, it is hardly possible to compare the different approaches for dynamic scenarios. On the one hand, each approach deals with different scenarios [1]: regards the case of vehicle capacity of 1, which leads to some kind of TRP [2] has no net but coordinates, [3] does not allow a replanning of a truck with an order, [4] does not allow a transhipment of packages and [5] takes stochastic information into account (which is an improvement here). Due to this large diversity of dynamic problem formulations, we decided to compare our new approach to the most basic VRP instances in this field. An additional advantage of this approach is that we were able to compare our approach to traditional algorithms and to an absolute scale with the optimal solution—which normally cannot be calculated for the dynamic scenarios.

The approach to the transportation problem taken for the DLRP is basically different to the approach for the traditional VRP. The developed protocol is not an optimisation algorithm for a static scenario, but an autonomous control algorithm, designed for a continuous changing process. In order to compare both concepts, it is necessary to execute them on one scenario which both sides can cope with.

To draw such an adapted vehicle routing problem for both sides, let us first have a look at major differences between the traditional vehicle routing problem formulation and the scenarios for the DLRP:

- the VRP instances have only coordinates so every possible path is allowed—the DLRP scenario has few edges and no fully connected net
- the DLRP is designed for a dynamic scenario, orders may appear at every vertex and at every time—the VRP orders are known from the beginning
- the VRP enforces closed routes—because of its dynamic control nature, the DLRP has ongoing, unclosed routes
- the objective function for the most VRP instances is the sum of all vehicle distances—within the DLRP, all

objects have their own objective function, e.g. shortest way for packages and the best utilisation for vehicles

It is not reasonable for a DLRP implementation to deal with full nets. In full nets, the RouteRequest/RouteReply-mechanism leads to a factorial growing number RouteRequest objects—a combinatorial explosion. Because it is easy for most traditional VRP-algorithms to build them for not-full-nets, we decided to restrict the net to feasible edges for the adapted problem.

We decided to take a real world network. In Fig. 2 you can see the chosen topology, which is the basic autobahnnet of Germany. The topology contains 18 vertices, the biggest cities in Germany, and 35 undirected edges. The scenario edge lengths match the real ones.

All traditional VRP optimisation algorithms have a static nature. They can only handle dynamic environments, if they are embedded in a replanning algorithm. On the other side, the DLRP can cope with these static cases, even though it was created especially for the control of dynamic environments. For a good performance, the DLRP has to be adopted for this special static case. Hence the adapted VRP was created as a static problem, all orders are known from the beginning.

In the DLRP, the routes of the vehicles are never-ending. Because of this character, it is almost impossible to find the point where a vehicle has finished its work within a bounded scenario. On the other hand, it is not too difficult for the traditional VRP-algorithms, to transfer the objective function from closed to non-closed vehicle routes. So the adapted VRP treats the distance of non-closed vehicle routes as objective to minimise.

The fourth point is not a conflict, but needs to be mentioned. The original [12] as well as the most discussed VRP-formulations (e.g. [13]) take the sum of driven vehicle distances as objective to minimise—other objective functions are also discussed, see e.g. [1, p. 276]. But for the traditional objective, we can easily show that the overall vehicle distance  $(\sum d_j)$ , package distances  $(p_i)$  and vehicle utilisation  $(u_i)$  are connected:

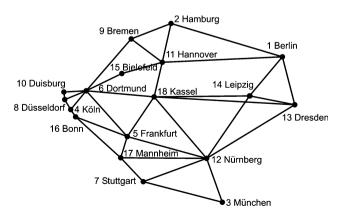


Fig. 2 Topology for the adapted VRP (from [11])



$$\frac{\sum_{\text{all packages } i} p_i}{\sum_{\text{all vehicles } j} u_j \cdot \frac{d_j}{\sum_{d_j} d_j}} = \frac{\text{vehicle capacity}}{\text{package size}} \cdot \sum_{\text{all vehicles } j} d_j \qquad (1)$$

This equation requires uniform vehicles and uniform packages concerning capacity and size. So the DLRP will indirectly minimise the requested objective function even though it primarily minimises the package distances and maximises the vehicle utilisation.

The new adapted VRP scenarios were built like distribution scenarios in this first application. All vehicles start at a central vertex, the city of Kassel, and all orders start form there. In order to keep the optimum value computable, the size of the scenarios is not too large. The number of vehicles can be 3, 6 or 9, while the number of packages can vary between 17, 34, 51 or 68. In addition we created a large scenario with 68 packages and 12 vehicles, which was the largest optimal solvable scenario. The amount of packages was matched to the topology. All 17 vertices, except Kassel, were supposed to be costumers. The package destinations for the larger scenario sets are uniformly distributed, whereas each vertex has one package at least. Therefore the 34, the 51 and the 68-scenarios have 10 subsets with different package destinations. The vehicle capacity was chosen in that way, that every vehicle is needed, if no vehicle comes back to Kassel. All different scenarios are shown in Table 1.

For an overview, three indicator values can be given:

- the shortest way from Kassel via all 17 vertices is 2,235 km long
- the shortest way without any loop is 2,245 km long
- the sum of the direct shortest ways for each of the 17 packages is 4,965 km

For a more detailed description of the scenarios concerning the topology distances and the distribution of the package destinations, feel free to contact the authors or refer to [14]. Tables and scenario data can be found on "http://dlrp.biba.uni-bremen.de".

Table 1 Chosen scenarios and corresponding vehicle capacities

		number of vehicles				
		3	6 9		12	
	17	8 pck/veh	3 pck/veh	2 pck/veh	-	
number of packages	17	,				
	34	16 pck/veh	6 pck/veh	4 pck/veh	-	
		10 subsets				
	51	25 pck/veh	10 pck/veh	6 pck/veh	-	
l mpe	31	10 subsets				
⊒	68	33 pck/veh	13 pck/veh	8 pck/veh	6 pck/veh	
	00	10 subsets				



## 4.1 Random like solution

In order to give something like an upper bound for reasonable solutions to the scenarios, a random-like heuristic was implemented.

In the first step, all packages are assigned randomly to the vehicles. A uniform distribution is used for this step with the restriction that every vehicle has to carry at least one package. The second step is to find an optimal way for one vehicle and its load of packages. To save computing time, only loop-free routes are considered. This restriction makes the route non optimal in some cases, but it is assumed that the optimal solution is not too far (see above, the difference between the shortest way with and without loops is 10 km or 0.4%).

This algorithm was calculated 10,000 times for each subset. The resulting mean values are shown in the next table (Table 2).

The described algorithm has some analogies with the real world transport market: the different forwarder companies receive their orders randomly and each company tries to optimise its vehicle routes on its own. An overall optimum cannot be expected from a procedure like this. Note that the average utilization which was reached by the DLRP in the largest scenario 68-12, about 70%, would be a very good value for real world forwarder companies. The vision of the DLRP is to implement this protocol independently from different companies. In this vision, an overall optimisation can happen without taking any decision possibilities from the single forwarders [8].

## 4.2 Optimal solution

On the other side, a lower bound for the overall vehicle distance should be given. To calculate optimal solutions for the given instances, the specified problem was formulated as a mixed-integer program (MIP, see below). The objective of the program is based on the formulation of Dantzig and Ramser [12] and minimizes the distance driven by all vehicles. In order to cope with the characteristics described

Table 2 Vehicle distances calculated by the random-like algorithm

Number of packages	Number of vehicles				
	3	6	9	12	
17	3,927	5,118	5,474		
34	5,051	7,594	9,072		
51	5,583	8,940	11,225		
68	5,943	9,897	12,756	14,876	



Table 3 Optimal vehicle distances

Number of packages	Number of vehicles			
	3	6	9	12
17	2,245	2,565	3,095	
34	2,245	2,868	3,310	
51	2,245	2,608	3,362	
68	2,245	2,683	3,400	4,097

in chapter 3, we adapt the formulation of the VRP. The proposed formulation was implemented in GAMS and could be solved with CPLEX 11. Because of the quite small size of the instances, the MIP could be solved within acceptable time—the scenario 68-12 seems to be the largest one which can be reasonably solved optimal.

Table 3 shows the optimal values for the overall vehicle distances. Note that in this solution, routes with loops are allowed and each vehicle has to be used.

## 4.2.1 MIP-formulation

## 4.2.1.1 Nomenclature

#### Sets

I vertices of the network

 $I^D$  depots of the considered network ( $I^D \subset I$ ); in our case only one depot exists

 $I_i^S$  vertices that are directly connected to vertex  $i(I_i^S \subseteq I)$ 

S segments of vehicle routes

V vehicles

#### **Parameters**

 $c_{i,i'}$  distance between vertex i and i'

 $d_i$  demand of packages at vertex i

 $p_i$  provided packages at vertex i; in our case packages are only provided at the depot  $i \in I^D$  with  $p_i = \sum_{i \in I} d_i$ 

 $\bar{r}_v$  transportation capacity of vehicle v

r required transportation capacity of one package

M a very large number

#### Variables

 $u_{v,i,i',s}$  amount of transported packages from vertex i to i' by vehicle v in s

 $x_{v,i,i',s}$  binary variable denoting that vehicle v drives in s from vertex i to i'

 $z_{v,i,s}$  binary variable denoting that vertex *i* is the end of the route of vehicle *v* in *s* 

4.2.1.2 Model assumptions The applied formulation of the vehicle routing problem has characteristics of the

capacitated vehicle routing problem (CVRP) and the split order vehicle routing problem (SDVRP). Within the considered network only one depot exists and the vertices are connected by undirected edges. Thus the associated distance between two directly connected vertices is the same in either way. The demand of each vertex is a priori known. In this context the demand of a certain vertex can be split and met by either one or multiple deliveries. According to the CVRP the capacity of the vehicles is limited. For our analysis we assume the same capacity  $\bar{r}_v$  for every vehicle v. All vehicles start their route at the depot and have to be used. In our modeling approach the route of a vehicle is described by a number of consecutive segments. In this way a segment represents either a movement of a vehicle from vertex i to i' or the end of the route. Note that the total number of permitted segments of a route is a critical constraint of the problem. For our analysis we have chosen the number of segments as high as the number of vertices within the network. Every vehicle is allowed to visit each vertex several times during its route. This characteristic of the model permits a vehicle on the one hand to serve a remote vertex from a given vertex and to return afterwards to the vertex before it continues its route through the network and on the other hand to pick up packages several times from the depot. Furthermore each vertex can be used to store packages. This means a vertex can receive more packages than requested and that these packages can be picked up and delivered to other vertices. At the end of their route the vehicles remain at the vertex of their final delivery and do not have to return to the depot.

4.2.1.3 Mathematical model Problem constraints: The first segment of the route of each vehicle starts at the depot.

$$\sum_{i' \in I_s^s} x_{v,i,i',s} = 1 \quad (i \in I^D; \ v \in V; \ s = 0)$$
 (2)

The route of each vehicle can be terminated only once at any vertex of the network.

$$\sum_{s \in S} \sum_{i \in I} z_{v,i,s} = 1 \quad (v \in V)$$
(3)

In every segment the route of a vehicle can be either continued or terminated.

$$\sum_{i \in I} \sum_{i' \in I_s^S} x_{\nu,i,i',s} + \sum_{i \in I} z_{\nu,i,s} \le 1 \quad (s \in S; \ \nu \in V)$$
 (4)

The route of a certain vehicle is described by a set of consecutive segments. This means a vehicle has either to leave its current vertex in the successive segment or to terminate its route. The introduction of segments ensures that a vehicle can visit a certain vertex several times and no



independent sub cycles occur. Note that the depot can be visited more than once as well.

$$\sum_{\substack{i \in I: \\ h \in I_i^S}} x_{\nu,i,h,s} - \sum_{i' \in I_h^S} x_{\nu,h,i',s'} - z_{\nu,h,s'} = 0$$

$$(h \in I; s, s' \in S: s' = s + 1; \nu \in V)$$
(5)

The load of each vehicle has to be less than or equal to the maximum transportation capacity.

$$u_{v,i,i',s}r \le \bar{r}_v \quad (i \in I; \ i' \in I_i^S; s \in S; \ v \in V) \tag{6}$$

Within the network the packages are transported by the vehicles between directly connected vertices. In this context every vertex can receive, store and ship packages. Furthermore each vertex has a deterministic demand of packages. In our case only the depot  $i \in I^D$  has a stock of packages  $p_i$  in the beginning.

$$p_{i} + \sum_{v \in V} \sum_{\sigma=0}^{s} \sum_{\substack{h \in I: \\ i \in I_{h}^{s}}} u_{v,h,i,\sigma} = d_{i} + \sum_{v \in V} \sum_{\sigma=0}^{s} \sum_{i' \in I_{i}^{s}} u_{v,i,i',\sigma}$$

$$(i \in I; s \in S)$$
(7)

Packages can only be shipped between two directly connected vertices i and i' if a vehicle v serves this segment s on its route. Equation 8 simplifies the problem in a way that avoids a formulation as a mixed integer non-linear program.

$$u_{v,i,i',s} \leq x_{v,i,i',s} M \quad (i \in I; \quad i' \in I_i^S; \ s \in S; \ v \in V)$$
 (8)

Objective function: The objective of the formulation is to minimize the distance driven by all vehicles.

Min. 
$$\sum_{v \in V} \sum_{s \in S} \sum_{i \in I} \sum_{i' \in I_s^S} x_{v,i,i',s} c_{i,i'}$$
 (9)

### 4.3 Tabu search solution

As a representative of established solution techniques (see [1, 15]) for vehicle routing problems, a tabu search algorithm was applied to the scenarios.

This algorithm is similar to the random-like solution technique. One solution set for the tabu search is one assignment set. This set assigns all packages to the vehicles. After the assignment, an optimal route or each vehicle is calculated (again with loop-free routes only like the random-like algorithm).

For the neighbourhood generation, the  $\lambda$  -interchange generation mechanism by Osmand [16] was implemented. The  $\lambda$  was set to 1 and insured that no vehicle is empty. The maximum age of elements within the tabulist was set to 6 and the search was aborted after 18 moves. Note that tabu search is very close to the optimal solution in the small scenarios and moves away with greater scenarios (Table 4).



Number of packages	Number of vehicles			
	3	6	9	12
17	2,410	2,610	3,095	
34	3,335	3,689	3,775	
51	3,988	4,637	5,026	
68	4,409	5,782	6,580	7,058

#### 4.4 DLRP solution

The DLRP solution was calculated with the DLRP roughly described above. To increase the performance, the protocol was adapted to this special static situation. Additionally the decision functions for the different objects were simplified and harmonized: packages only choose their routes by the route length, vehicles only by the estimated utilisation. This means that the packages do not adjust to the vehicle routes, in contrast to the dynamic version. Only vehicles choose their route dependent on the package route situation.

The results are shown in Table 5. Routes with loops are allowed here and each vehicle does not need to carry a package, but do so.

#### 5 Conclusions

For a better overview, all results are shown as line charts in the following Figs. 3, 4, 5 and 6. Compared to the optimum and the random-like solution, the tabu search heuristic leads to near optimal results with small scenarios and gets worse with larger scenarios. This is a normal behaviour for tabu search algorithms and is due to the exponential growing solution space.

In contrast, the DLRP solutions are not very good with small scenarios, but get better with larger scenarios. The large scenarios 68-6, 68-9 and 68-12 show that the DLRP gets better than the tabu search concept with a growing network size.

Because the DLRP is originally a control method, its main advantages point to dynamic and close to reality scenarios: self-adaptation to changing situations, possible

Table 5 Vehicle distances calculated by the DLRP

Number of packages	Number of vehicles			
	3	6	9	12
17	4,530	2,875	4,400	
34	5,184	5,928	7,022	
51	4,918	4,807	6,310	
68	4,558	5,586	6,331	6,550



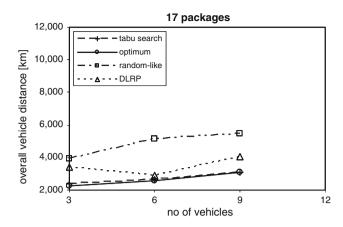


Fig. 3 Results for 17 packages

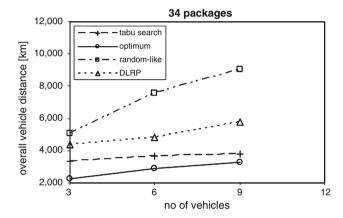


Fig. 4 Results for 34 packages

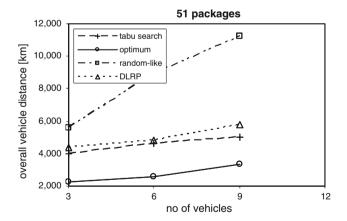


Fig. 5 Results for 51 packages

manual interventions at runtime, implementation of uncertain knowledge and complex and context driven decision functions [8]. The described results show that the DLRP has a high potential. Apart from the described advantages the DLRP can be coequal to classical VRP concepts in real world scenarios. DLRP results from *large* 

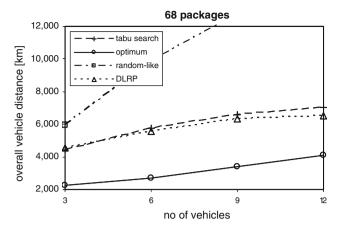


Fig. 6 Results for 68 packages

and *dynamic* scenarios with 2,500 packages emphasise this assumption. These results are shown in Fig. 7.

#### 5.1 New evaluation chart

For larger scenarios, the classification and evaluation of algorithms for the VRP or the adapted VRP gets more and more difficult. For these scenarios, it is not possible to calculate optimal solutions, so a lower bound is missing. Additionally, it is not possible to compare two different scenarios, because the optimal way lengths can be very different. On the basis of Eq. (1), we suggest an alternative comparison approach. The vehicle utilisation and the package distances are, with some restrictions, directly connected to the overall vehicle distance. The vehicles utilisation has a natural best value and upper bound: it cannot be greater than one. The package distances have a natural best value and lower bound, which is easy to calculate: their individual shortest way to their destination. Therefore we can define a relative package distance: driven package distance by shortest possible distance. The lower bound of this relative package distance is one.

Now we can illustrate these two values in one plane, exemplarily shown in Fig. 7. The upper right corner represents one theoretical extreme: each package has one vehicle of capacity 1 and drives its shortest way. The horizontal line at utilisation = 0.5 is another extreme: when one vehicle with capacity for all packages brings out all packages, the utilisation goes to 0.5 and the relative package distance increases infinitely.

In this suggested evaluation chart, on can see that an optimal solution must be somewhere in the upper right corner: a relative package distance of nearly 1 and a high utilisation of 80–90% (see optimum of scenario 68-12).

In Fig. 7, some of the larger scenario results are shown. With this kind of chart, it is possible to compare the large dynamic scenario results as well. Even though there are no



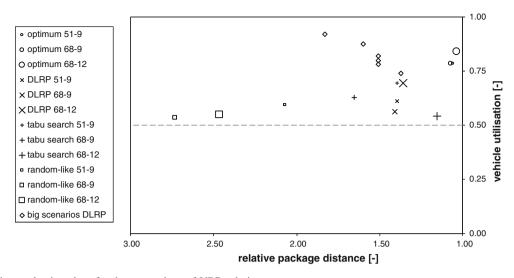


Fig. 7 Alternative evaluation chart for the comparison of VRP solutions

optimal or tabu search values for these scenarios, one can say that the DLRP works well with these large dynamic environments. Additionally, one can see that it is possible to change the DLRP parameters in a way that either the vehicle utilisation or the relative package distance is preferred.

Another practical advantage of this evaluation chart is that the suggested values can be treated as a state function. It is possible to measure the vehicle utilisation and the relative package distance continuously. Consequently it is possible to control these values.

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# References

- Vahrenkamp R, Mattfeld D (2007) Logistiknetzwerke. Gabler, Wiesbaden
- Fleischmann B, Gnutzmann S, Sandvoß E (2004) Dynamic vehicle routing based on online traffic information. Transp Sci 38(4):420–433
- Jaillet P, Wagner MR (2006) Online routing problems: value of advanced information as improved competitive ratios. Transp Sci 40(2):200–210
- Hiller B, Krumke SO, Rambau J (2006) Reoptimization gaps versus model errors in online-dispatching of service units for adac. Discrete Appl Math 154:1897–1907
- Savelsbergh M, Sol M (1998) Drive: dynamic routing of independent vehicles. Oper Res 46:474

  –490

- Bent RW, Van Hentenryck P (2004) Scenario-based planning for partially dynamic vehicle routing with stochastic customers. Oper Res 52(6):977–987
- 7. Perkins CE (2001) Ad hoc networking. Addison-Wesley, Boston
- Scholz-Reiter B, Rekersbrink H, Freitag M (2006) Internet routing protocols as an autonomous control approach for transport networks. In: Proceedings of the 5th CIRP international seminar on intelligent computation in manufacturing engineering, pp 341–345
- 9. Scholz-Reiter B, Rekersbrink H, Freitag M (2006) Kooperierende Routingprotokolle zur Selbststeuerung von Transportprozessen. Industrie Management 22/3, pp 7–10
- Wenning B-L, Rekersbrink H, Timm-Giel A, Görg C, Scholz-Reiter B (2007) Autonomous control by means of distributed routing. In: Understanding Autonomous Cooperation & Control in Logistics—The Impact on Management, Information and Communication and Material Flow. Springer, Berlin, pp 325–336
- Wenning B-L, Rekersbrink H, Becker M, Timm-Giel A, Görg C, Scholz-Reiter B (2007) Dynamic transport reference scenarios.
   In: Understanding autonomous cooperation & control in logistics—the impact on management, information and communication and material flow. Springer, Berlin, pp 337–350
- Dantzig GB, Ramser JH (1959) The truck dispatching problem. Manage Sci 6(1):80–91
- Solomon MM (1987) Algorithms for the vehicle routing and scheduling problems with time window constraints. Oper Res 35(2):254–265
- 14. Scholz-Reiter B, Rekersbrink H, Wenning, B-L, Makuschewitz T (2008) A survey of autonomous control algorithms by means of adapted vehicle routing problems. In: Proceedings of the 9th Biennial ASME conference on engineering systems design and analysis ESDA 08 (on CD), Haifa, Israel
- Laporte G (1992) The vehicle routing problem: an overview of exact and approximative algorithms. Eur J Oper Res 59:345–358
- Osman IH (1993) Metastrategy simulated annealing and tabu search for the vehicle routing problem. Ann Oper Res 41:421– 451

