CIRP Annals - Manufacturing Technology 60 (2011) 485–488

Contents lists available at ScienceDirect

CIRP Annals - Manufacturing Technology

journal homepage: http://ees.elsevier.com/cirp/default.asp

Assessment of fidelity of control-theoretic models of WIP regulation in networks of autonomous work systems

V. Toshniwal [a], N. Duffie (1)[b,*], T. Jagalski [c], H. Rekersbrink [c], B. Scholz-Reiter (2)[c]

[a] Manufacturing Systems Engineering Program, University of Wisconsin-Madison, WI, USA
[b] Department of Mechanical Engineering, University of Wisconsin-Madison, WI, USA
[c] Department of Planning and Control of Production Systems, University of Bremen, Bremen, Germany

ARTICLE INFO

Keywords:
Production
Control
Dynamics

ABSTRACT

One means of adapting to variation in demand is making capacity flexible so that work in progress (WIP) can be regulated; however, this can significantly influence the dynamic behavior of production networks in which there is high local autonomy. Control-theoretic models are a convenient means for investigating and designing the dynamics of such networks, but the fidelity of these models is not well understood. In this paper, results obtained using discrete-event simulations are used to assess the control-theoretic approach, providing evidence that fidelity varies depending upon factors such as WIP level and the magnitude of capacity adjustments.

1. Introduction

Industries are being continually challenged by variability due to product variety, shortened lead times, rush orders, etc. Industries have attempted to address these issues using concepts such as agile manufacturing, quick response manufacturing and lean manufacturing. Centralized control and information sharing has been recommended to reduce effects of variability in demand in production networks and unfavorable dynamic behavior such as the 'bullwhip effect' [1]. However, required information sharing and complexities due to growing numbers of echelons and relationships in networks complicates centralized control, and autonomous decentralized control has been recommended to better respond to changing markets [2,3]. As production networks expand and increase in complexity, it is important to ensure that local decision making and disturbances do not adversely affect their dynamic behavior [4].

Techniques and tools of control theory can be used to understand dynamics of production networks. A review of research in this area suggested that control theory can be used to reduce inventory variations, demand amplifications and optimize order release rules in the networks [5]. Industries often struggle with maintaining optimal work in progress (WIP) to satisfy conflicting objectives of short lead times and high utilization when there is high variability in demand [6]. Non-linear operation rules have been developed to adjust WIP in complex networks [7]. Control-theoretic models have been used to analyze stability of production networks [8], and recommended for regulating lead times and improving customer service [9]. Control-theoretic approaches have been proposed for WIP regulation to improve operating performance, and dynamic models have been developed for WIP regulation in networks of autonomous work systems [10–12].

Control-theoretic models of production networks can be developed more quickly than discrete-event simulation models, and produce estimates of fundamental dynamic properties such as time constants and damping ratios that characterize how rapidly production networks respond to turbulence and whether responses are oscillatory. However, the fidelity with which control-theoretic models predict the fundamental dynamic behavior of production networks is not well understood and has not been assessed. Many assumptions are required to make control-theoretic models tractable, and many details are ignored regarding the logistics of production. Such an assessment is described in this paper. A specific industrial scenario was used to assess the fidelity of a control-theoretic model of a network of autonomous work systems with local WIP regulation. A discrete-event simulation was developed for the same scenario and used as the benchmark for comparison. In the following sections, these models are described and areas of agreement and deviation are identified. The benefits of WIP regulation using a control-theoretic approach are also discussed.

2. Control-theoretic model

It was assumed in the control-theoretic model that was studied that work system capacity can be periodically adjusted. N work systems were assumed to be present in the production network. Setup times, transportation times and capacity and buffer limitations were not considered. A constant delay of d days was assumed in implementing change in capacity due to labor or other issues. Inputs were assumed to be constant during time $kT \leq t < (k+1)T$, where $k = 0, 1, 2, \ldots$ and T is the time period between capacity adjustments. Input to the work systems at time $(k+1)T$ was represented using the vector [12].

$$\mathbf{W}_i((k+1)T = \mathbf{W}_i(kT) + T(\boldsymbol{i}(kT) + \mathbf{P}^T\mathbf{C}_a(kT)), \quad (1)$$

* Corresponding author.

0007-8506/$ – see front matter © 2011 CIRP.
doi:10.1016/j.cirp.2011.03.045

where $\mathbf{i}(kT)$ and $\mathbf{C}_a(kT)$ represent input and output rates of the work systems, respectively. $\mathbf{P}$ is a workflow matrix in which each element p_{nj} represents the fraction of work flowing from work system n to work system j during time $kT \le t < (k+1)T$. Total output from the work systems until time $(k+1)T$ is

$$\boldsymbol{W}_o((k+1)T) = \boldsymbol{W}_o(kT) + \boldsymbol{C}_a(kT)T \tag{2}$$

The WIP is

$$\boldsymbol{WIP}_a(kT) = \boldsymbol{W}_i(kT) - \boldsymbol{W}_o(kT) + \boldsymbol{W}_d(kT), \tag{3}$$

where $\mathbf{W}_d(kT)$ represents disturbance inputs to the work systems such as rush orders. If the WIP in each work system is desired to be maintained at a planned level $\mathbf{WIP}_p(kT)$, then the change in capacity $\Delta\mathbf{C}(kT)$ at the work systems can be adjusted with respect to planned capacity $\mathbf{C}_p(kT)$ using a simple control law with proportional control constant K_c:

$$\Delta\boldsymbol{C}(kT) = K_c(\boldsymbol{WIP}_a(kT) - \boldsymbol{WIP}_p(kT)) \tag{4}$$

The actual capacity (production rate) $\mathbf{C}_a(kT)$ is

$$\boldsymbol{C}_a(kT) = \boldsymbol{C}_p(kT) + \Delta\boldsymbol{C}((k-d)T) + \boldsymbol{C}_d(kT) \tag{5}$$

where $\mathbf{C}_d(kT)$ represents disturbances in capacity.

The primary limitations of the control-theoretic model were:

- Individual orders and machines were not represented.
- Capacity and WIP could vary outside practical limits.
- Set up and transportation times were not represented.
- A constant work-flow matrix was used.

3. Discrete-event simulation

3.1. Without WIP regulation

To assess the fidelity of the control-theoretic model, a discrete-event simulation (DES) model was developed using the commercial software ARENA, and an industrial dataset was used as the simulation scenario. This dataset was from a supplier to the automotive industry and it documented 659 orders that were processed in a 186-day period [13]. In the simulation, machines were grouped to form five work systems: Shearing/Sawing, Ring Rolling, Drop Forging, Heat Treatment and Quality Control as shown in Fig. 1. WIP was measured in hours of work.

Limitations of the DES model included:

- Labor was not considered as a resource.
- Set up and transportation times were not considered.
- Machine failures and downtime were not considered.
- Work on weekends was not considered.

The time when each order entered into the network and the service times at each work system were documented in the dataset and used in the simulation. The capacity of each work system was fixed at the average daily input in the dataset. Fig. 2 shows the WIP

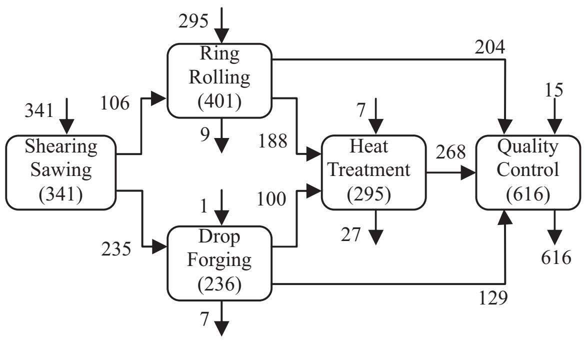

Fig. 1. Flow of orders in the network of work systems that was studied.

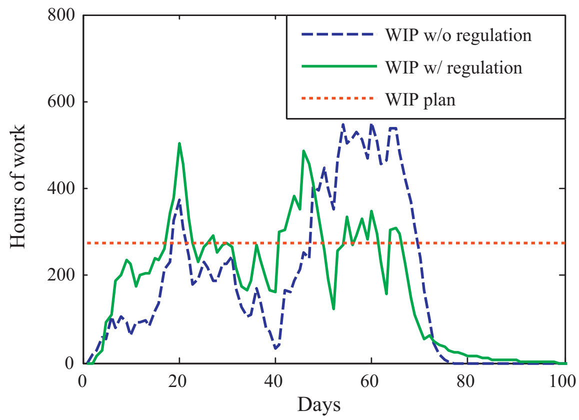

Fig. 2. WIP in the Drop Forging work system predicted by DES with and without WIP regulation.

Table 1
Mean WIP and variation in WIP with and without WIP regulation.

Work system	DES w/o WIP regulation		DES w/WIP regulation		% decrease in σ
	Mean	σ	Mean	σ	
Shearing/Sawing	153	52	153	41	23%
Ring Rolling	370	142	381	77	45%
Drop Forging	273	159	279	83	48%
Heat Treatment	110	59	116	51	14%
Quality Control	344	98	377	70	28%

Table 2
Mean lead time and variation in lead time with and without WIP regulation.

Work system	DES w/o WIP regulation		DES w/WIP regulation		% decrease in σ
	Mean	σ	Mean	σ	
Shearing/Sawing	54	36	54	31	16%
Ring Rolling	74	49	74	42	14%
Drop Forging	106	61	105	44	27%
Heat Treatment	53	40	49	34	15%
Quality Control	66	74	58	62	16%

predicted by this DES model for the Drop Forging work system. Variation in WIP was similar in the other work systems.

3.2. With WIP regulation

Eqs. (4) and (5) were added to the DES to calculate capacity adjustments for the work systems for the purpose of regulating WIP. These adjustments were implemented by changing work-day length rather than the number of operating machines. The WIP predicted using $K_c = 0.25\ \text{day}^{-1}$ in the Drop Forging work system is shown in Fig. 2. Average WIP found using DES without WIP regulation was used as the planned WIP, and average daily input also found using DES without WIP regulation was used as the planned capacity for each work system. The ramp up and ramp down periods were excluded in calculating averages. Table 1 shows that variation in WIP was reduced in all of the work systems by WIP regulation. Table 2 shows that variability in lead time was intrinsically reduced, even though service times varied significantly between orders, thus improving on-time deliveries and customer service.

4. Control-theoretic simulation

A control-theoretic simulation (CTS) model was implemented in Excel using Eqs. (1) through (5). Internal flow of orders was approximated using the workflow matrix shown in Table 3. These data were obtained from the DES without WIP regulation. WIP at the Shearing/Sawing work system predicted by CTS was the same

Table 3
Workflow matrix (fractions in hours/hour).

From	To				
	Shearing/Sawing	Ring Rolling	Drop Forging	Heat Treatment	Quality Control
Shearing/Sawing	0	2352/3192	4647/3192	0	0
Ring Rolling	0	0	0	2488/7785	2308/7785
Drop Forging	0	0	0	1264/4652	1504/4652
Heat Treatment	0	0	0	0	3650/4012
Quality Control	0	0	0	0	0

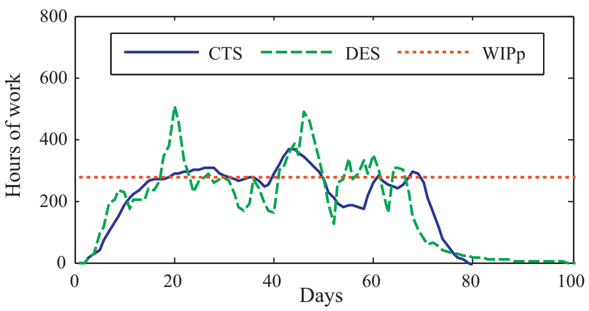

Fig. 3. WIP at the Drop Forging work system predicted by CTS and DES with WIP regulation.

as that predicted by DES with WIP regulation because it received no work from other work systems. Fig. 3 shows WIP predicted by CTS for the Drop Forging work system along with WIP predicted by DES with WIP regulation.

5. Assessment of fidelity

The control-theoretic model had significant limitations that affected prediction of the fundamental behavior of the autonomous work systems. The fidelity of control-theoretic model was assessed by comparing its results to DES with WIP regulation and considering factors such as response to disturbances, oscillatory response, behavior at low WIP, and use of constant work flow matrix **P** in Eq. (1).

5.1. Response to disturbances

Response of WIP to work disturbances predicted by control theory and DES were compared and settling times were measured. Work flow was unidirectional in this network, and the control-theoretic transfer function for change in WIP at work system i for a work disturbance at work system i is

$$\frac{\Delta WIP_i(z)}{W_i(z)} = \frac{(1-z^{-1})z^{-1}}{1-z^{-1}+K_c T z^{-(d+1)}} \tag{6}$$

Fig. 4 shows the change in WIP predicted by Eq. (6) and DES for $T = 1$ day, $d = 1$ day, and $K_c = 0.25$ for a rush order of 90 h of work at the Shearing/Sawing work system with $WIP_p = 376$ h. Change in WIP for DES was calculated by subtracting daily WIP without the rush order from daily WIP with the rush order. Time is relative to

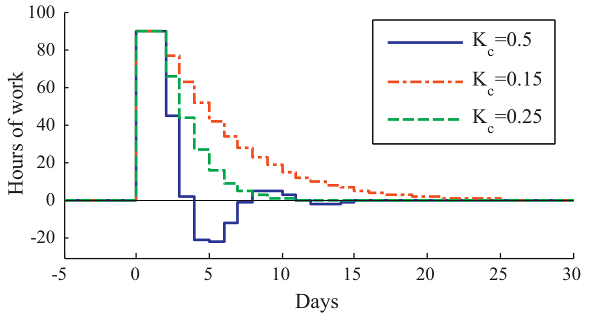

Fig. 4. Response to a rush order predicted by CTS and DES with WIP regulation.

the arrival of the rush order, which was variable in the DES. Change in WIP predicted by control theory and change in WIP predicted by DES were nearly identical for all work systems.

5.2. Oscillatory response

Using control-theoretic analysis with $T = 1$ and $d = 1$ day, the roots of the denominator of Eq. (6) become complex for $K_c > 0.25$, predicting oscillatory behavior in the work system. Fig. 4 also shows the change in WIP predicted by control theory and DES at the Shearing/Sawing work system with gains of $K_c = 0.5$. The predicted oscillatory response was nearly identical in both the models, as was response with $K_c = 0.15$. Results of significantly more oscillatory response at higher gains, $K_c > 0.7$, deviated because capacity and WIP were not limited in the control-theoretic model. Fidelity of prediction of response to work disturbances therefore was good for practical values of K_c.

5.3. Behavior at low WIP

The lack of representation of individual machines and orders in the control-theoretic model had a significant impact on predicted capacity, utilization and settling time at low WIP. The DES results in Fig. 5 show that WIP regulation adjusted full capacity to higher average levels at lower planned WIP (results were similar for all work systems) because the number of machines did not change during capacity adjustment, and machines deprived of orders remained idle, reducing utilization. These results are analogous to logistic operating curves proposed by Nyhuis [6]. The control-theoretic model does not predict lower utilizations. Fig. 6 shows that average settling times for work disturbances with $K_c = 0.25$ were shorter at lower WIP. Rush orders were processed faster because spare capacity was available at the work systems due to lower utilization. The control-theoretic model predicted constant settling times regardless of WIP level.

5.4. Constant work flow matrix

The DES modeled the flow of orders from one work system to another based on individual routings and service times, whereas a constant work-flow matrix was used in the CTS to predict the input to downstream work systems. This matrix was calculated using average work-flow data, neglecting daily changes in the flow structure of orders.

Fig. 3 shows that the CTS and DES models predicted significantly different WIP variations at the Quality Control work

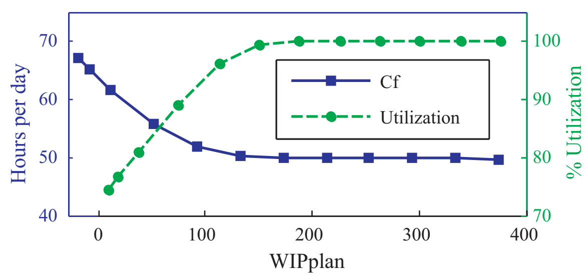

Fig. 5. Average full capacity (capacity without disturbances) and average utilization at the Shearing/Sawing work system with WIP regulation.

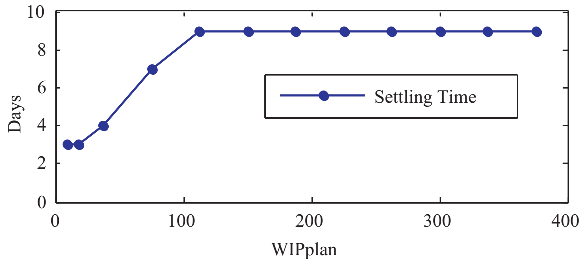

Fig. 6. Average settling time versus planned WIP.

Table 4
Variation of WIP with WIP regulation predicted by CTS and DES compared to DES without WIP regulation.

Work system	CTS	DES	Difference
Shearing/Sawing	22%	23%	−1%
Ring Rolling	63%	45%	18%
Drop Forging	72%	48%	24%
Heat Treatment	59%	14%	45%
Quality Control	62%	28%	35%

system. Table 4 shows the percent reduction in variation of WIP using WIP regulation as predicted by CTS and DES, both compared to DES without WIP regulation. The greater differences are downstream due to lack of modeling of variation in order service times and order routing in the CTS.

6. Conclusion

Discrete-event simulation models and an industrial dataset were used in this work to assess the fidelity of control-theoretic models. Except at low levels of WIP or when response was extremely oscillatory, the response of WIP regulation to work disturbances such as rush orders predicted by the control-theoretic models was nearly identical to that predicted using discrete-event simulation. Results of discrete event simulation of WIP regulation at low WIP showed lower utilization, higher capacities and lower settling times than predicted by the control-theoretic models because the latter do not represent individual orders and machines. Also, there was more variation in WIP with discrete-event simulation than with control-theoretic simulation because the latter neglected daily changes in work flow structure.

It was concluded that the fidelity of the control-theoretic model decreased at extreme conditions such as low WIP and large capacity adjustments at very high gain K_c, but predictions of fundamental dynamic behavior using transfer functions were otherwise good. Control-theoretic simulations of operation at extreme operating conditions were of significantly lesser fidelity than discrete-event simulations; thus, further research is required to improve their fidelity. Research already conducted on order-flow information sharing between autonomous work systems [12] may serve as a starting point in this regard.

Comparison of discrete-event simulation results with and without WIP regulation indicated significant reductions in variation of WIP from planned levels. More extensive simulation studies using distributions for arrival rates, service times, etc. are needed to confirm these results in a broader range of scenarios.

Acknowledgements

This work was supported in part by U.S. National Science Foundation grant DMI-0646697 and German Research Foundation grants SFB 637/2-A6 and Br 933/16-1.

References

[1] Helo P (2000) Dynamic Modeling of Surge Effect and Production Rate Limitation in Supply Chains. *International Journal of Production Research* 38(17):4521–4533.
[2] Scholz-Reiter B, Windt K, Freitag M (2004) Autonomous Logistic Processes: New Demands and First Approaches. *Proc. 37th CIRP International Seminar of Manufacturing Systems*, Faculty Press, Budapest, Hungary, 357–362.
[3] Hulsmann M, Windt K (2007) Changing Paradigms in Logistics—Understanding the Shift From Conventional Control to Autonomous Cooperation and Control. in Hulsmann M, Windt K, (Eds.) *Understanding Autonomous Co-operation and Control in Logistics*. Springer, Berlin, pp. 1–16.
[4] Wiendahl H-P, Lutz S (2002) Production in Networks. *CIRP Annals – Manufacturing Technology* 51(2):573–586.
[5] Ortega M, Lin L (2004) Control Theory Applications to the Production Inventory Problem: A Review. *International Journal of Production Research* 42(11): 2303–2322.
[6] Nyhuis P (2006) Logistic Operating Curves—Basic Model of the Theory of the Logistic Operating Curves. *CIRP Annals – Manufacturing Technology* 55(1): 441–444.
[7] Scholz-Reiter B, Freitag M, Schmieder F (2002) Modeling and Control of Production Systems Based on Non-linear Dynamics Theory. *CIRP Annals – Manufacturing Technology* 51(1):375–378.
[8] Disney SM, Towill DR (2002) A Discrete Transfer Function Model to Determine the Dynamic Stability of a Vendor Managed Inventory Supply Chain. *International Journal of Production Economics* 40(1):179–204.
[9] Towill DR, Evans GN, Cheema P (1997) Analysis and Design of an Adaptive Minimum Reasonable Inventory Control System. *Production Planning and Control Journal* 8(6):545–557.
[10] Duffie NA, Shi L (2009) Maintaining Constant WIP-regulation Dynamics in Production Networks With Autonomous Work Systems. *CIRP Annals – Manufacturing Technology* 58(1):399–402.
[11] Wiendahl H-P, Breithaupt J-W (1999) Modelling and Controlling the Dynamics of Production Systems. *Production Planning and Control Journal* 10(4):389–401.
[12] Duffie NA, Shi L (2009) Dynamics of WIP Regulation in Large Production Networks With Autonomous Work Systems. *IEEE Transaction on Automation Science and Engineering* 7(3):665–670.
[13] Toshniwal V. (2010) Assessment of WIP Regulation in Networks of Autonomous Work Systems, M.S. Thesis, Manufacturing Systems Engineering Program, University of Wisconsin-Madison.