Analysis of Lead-Time Regulation in an Autonomous Work System

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Abstract

The dynamic behavior of lead-time regulation in an autonomous work system is analyzed in this paper. A lead-time regulation topology is presented in which production rate is periodically adjusted to eliminate deviation that is caused by variation in the rate at which orders are input to the work system, as well as disturbances such as rush orders and equipment failures. A second, approximating topology is presented that is based on deviation of actual work output from desired work output and permits control-theoretic methods to be used to predict dynamic behavior and set control parameters. An example is presented to illustrate dynamic behavior, the equivalence of the topologies and the limitations of the lead-time deviation topology when production rates vary significantly.

Keywords:

Production, Control, Dynamic Model

1 INTRODUCTION

Over the years, researchers have proposed many approaches for reduce lead time and minimize its variability in production systems. In production planning, lead-time variability, and lead-time variance reduction in supply chains has been investigated along with its impact on performance in high-variety, low-volume production [2]. The effect of variability of order lead times and demand forecasting on the bullwhip effect in two-stage supply chains has been studied [3], and research has been conducted on the contributory factors for bullwhip effect, particularly focusing on variance amplification due to different forecasting algorithms [4]. The cost of lead-time variation has been studied [5], and lead-time variability reduction has been identified as a key factor in improving production systems [6].

Analogies have been established and tested for control theory and its applicability to production control [7]. Application of control theory to the production inventory problem has been reviewed [8], and the importance of feedback control in detecting and reducing inventory deviations has been examined [9]. State-space models have been used for switching between libraries of optimal controllers to adjust WIP in the presence of machine failures [10]. WIP regulation has been studied for large networks of autonomous work systems [11], and application of nonlinear dynamics theory has been investigated [12].

In this paper, a lead-time regulation topology is presented in which production rate is periodically adjusted to eliminate deviation between desired and actual lead time that is caused by variation in the rate at which orders are input to the work system and disturbances such as rush orders and equipment failures. A second, approximating topology also is presented that is based on deviation of actual work output from desired work output. This topology permits control-theoretic analysis methods to be used to predict dynamic behavior and set control parameters. An example then is presented to illustrate dynamic behavior, the equivalence of the topologies and the limitations of the lead-time deviation topology when production rates significantly vary.

2 LEAD-TIME REGULATION POLICY

Consider an autonomous work system in a production network that periodically adjusts its production rate to maintain local lead time $lt_a(kT)$ at a desired (planned) level lt_p, here assumed to be constant. T is the time period between these adjustments (for example, one shop calendar day (scd)), and k = 0, 1, 2, … The local production rate can be adjusted by first calculating the lead-time deviation

$$lt_e(kT) = lt_p - lt_a(kT) \tag{1}$$

and then using an appropriate policy to adjust the production rate with respect to a plan $c_p(kT)$. One such policy, shown in Figure 1, is

$c_f(kT) = c_p(kT) - k_{lt} lt_e\big((k-d)T\big)$. A lower value of k_{lt} tends to produce a slower reduction in lead-time deviation and, within limits, a higher value of k_{lt} tends to produce faster response. Adjustments in production rate are assumed to be delayed by time dT, integer d time periods, representing the realities of labor contracts and other logistic issues that prevent instantaneous adjustment of production rate. Plan $c_p(kT)$ is assumed to be known at least time dT in advance. Note that lead time is not minimized in this approach; rather, the goal is to maintain lead time at the desired level. Hence, production rate is decreased if the actual lead time is less than the desired lead time (the lead-time deviation is positive). In Figure 2, the following variables are assumed to be constant over time $kT \le t < (k+1)$:

- $i(kT)$ rate at which orders are input to the work system
- $w_d(kT)$ work disturbances such as rush orders and order cancellations
- $c_d(kT)$ production rate disturbances such as operator illness and equipment failures
- $c_a(kT)$ actual production rate

Furthermore, $w_i(kT)$ and $w_o(kT)$ represent the total orders input to and output from the work system, respectively, up to time kT. Production rate limits, buffer size limitations, setup times, transportation times, variations in delay with production rate adjustment magnitude, etc. are not modeled. Orders are used as the dependent variable rather than hours of work content, and the units of input and production rate therefore are orders/scd.

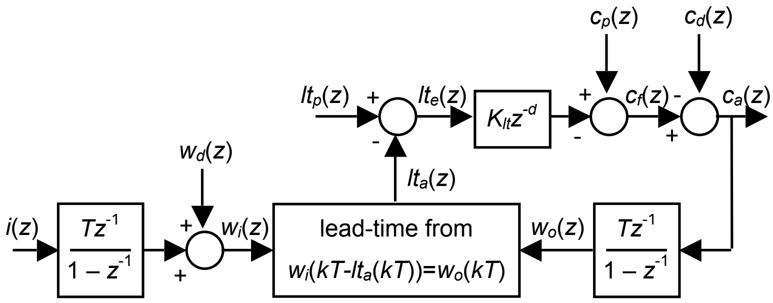

Figure 1: Lead-time regulation using a lead-time deviation topology.

3 DYNAMIC MODEL OF LEAD-TIME REGULATION

The fundamental dynamic properties of the lead-time deviation topology shown in Figure 1 are difficult to analyze using dynamic systems theory because measurement of lead time requires searching backwards in time to determine the time at which orders entered the work system. However, the topology in Figure 1 can be approximated using the topology shown in Figure 2 in which lead-time deviation is replaced by deviation in actual orders output $w_o(kT)$ from desired orders output $w_i(kT\text{-}lt_p)$:

$$w_e(kT) = w_i(kT - lt_p) - w_o(kT) \tag{3}$$

where desired lead time lt_p is assumed here to be an integer multiple of period T. Then, production rate is adjusted using

$$c_f(kT) = c_p(kT) + k_c w_e\big((k-d)T\big) \tag{4}$$

where k_c is a new control parameter. Figure 3 illustrates the relationship in time between work deviation and lead-time deviation.

From the model in Figure 2, transfer equations relating work deviation $w_e(z)$ and actual production rate $c_a(z)$ to the inputs $i(z)$, $w_d(z)$, $c_p(z)$ and $c_d(z)$ are

$$w_e(z) = \frac{Tz^{-\left(1+{lt_p}/{T}\right)} i(z) + \left(1 - z^{-1}\right) w_d(z) + Tz^{-1}\left(c_p(z) - c_d(z)\right)}{1 - z^{-1} + k_c T z^{-(d+1)}} \qquad (5)$$

$$c_a(z) = \frac{k_c T z^{-\left(d+1+{lt_p}/{T}\right)} i(z) + \left(1 - z^{-1}\right) k_c z^{-d} w_d(z) + \left(1 - z^{-1}\right)\left(c_p(z) - c_d(z)\right)}{1 - z^{-1} + k_c T z^{-(d+1)}} \qquad (6)$$

The fundamental dynamic properties of the work system then are described by the roots of

$$1 - z^{-1} + k_c T z^{-(d+1)} = 0 \qquad (7)$$

If it is desired to avoid both slow and oscillatory response, then k_c can be chosen using [11]

$$k_c = \frac{d^d}{(d+1)^{d+1}} \qquad (8)$$

The fundamental dynamic behavior described by Equation (7) for lead-time regulation using the work deviation topology in Figure 2 is not a function of production rate. However, this is not the case for the lead-time deviation topology shown in Figure 1 because the relationship between lead-time deviation $lt_e(kT)$ and work deviation $w_e(kT)$ shown in Figure 3 is dependent upon the rates at which work is input to and output from the work system. If there is a nominal rate $c_n(kT)$, then the control parameter k_{lt} in Figure 1 can be related to the control parameter k_c in Figure 2 by

$$k_{lt}(kT) = k_c c_n(kT) \qquad (9)$$

Therefore, if production rates change significantly with time, control parameter k_{lt} also may need to be correspondingly varied to maintain constant dynamic behavior.

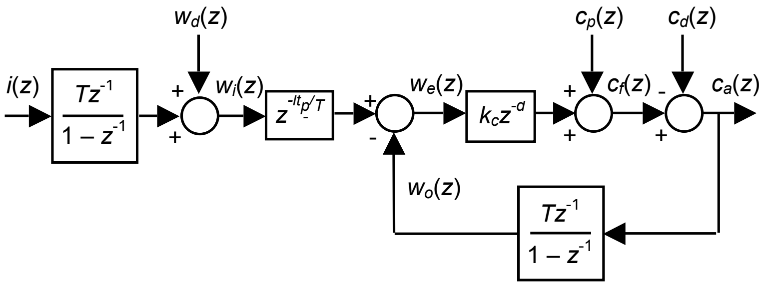

Figure 2: Lead-time regulation using a work deviation topology.

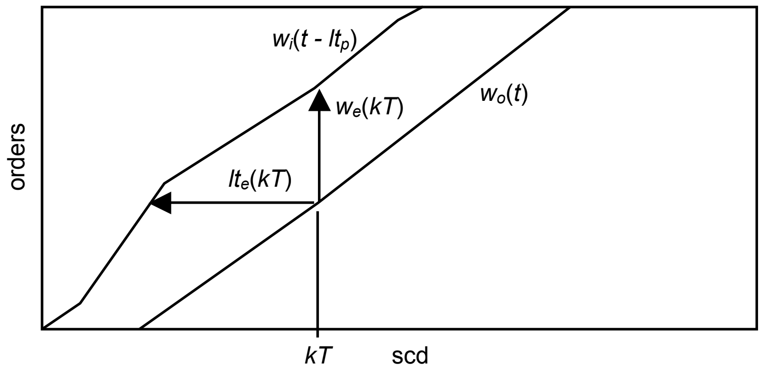

Figure 3: Relationship between lead-time deviation and work deviation.

4 EXAMPLE

Discrete-time simulation was used to predict work-system response for both lead-time regulation topologies. The input rate data shown in Figure 4 were used with T = 1 scd, d = 1 (delay dT = 1 scd), k_c = 0.25, and work and production rate disturbances were assumed to be zero. The planned capacity c_p was 5 orders/scd and k_{lt} = 1.25. Lead times were calculated at the beginning of each production rate adjustment period by finding integer j such that $W_i((k\text{-}j)T) \leq W_o(kT) < W_i((k\text{-}j+1)T)$ and interpolating in that interval. The resulting work in progress (WIP), lead time and actual capacity are shown in Figure 5 for the lead-time deviation topology. There is significant day-to-day variation in order input rate, and this leads to significant variation in WIP, while lead time variation is within approximately ±2 scd.

Figure 6 shows the lead-time deviation for regulation using both the lead-time topology and work deviation topology with the relationship between k_{lt} and k_c established by Equation (9) with c_n = c_p = 5 orders/scd. The results are very similar, providing evidence that the behavior of the model in Figure 2 and the control-theoretic analysis in Equations (5) through (7) are a good approximation in this case for the behavior of the model in Figure 1, even though input rate varies significantly with time.

On the other hand, Figure 7(a) shows the results for the lead-time deviation topology when the input rates are 4 times those shown in Figure 4, but k_{lt} is not adjusted according to Equation (9). The dynamics of lead-time regulation are significantly different in this case, and response is significantly slower. Conversely, Figure 7(b) shows the results for the lead-time regulation formulation when the input rates are 1/4 those shown in Figure 4, again without adjusting k_{lt}. In this case, the response has become noticably oscillatory.

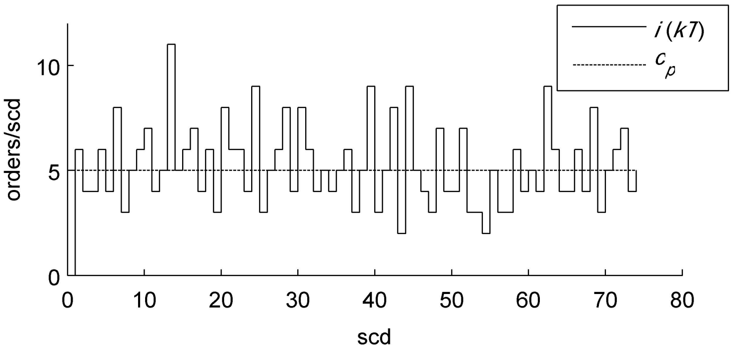

Figure 4: Input rate and planned production rate used in example.

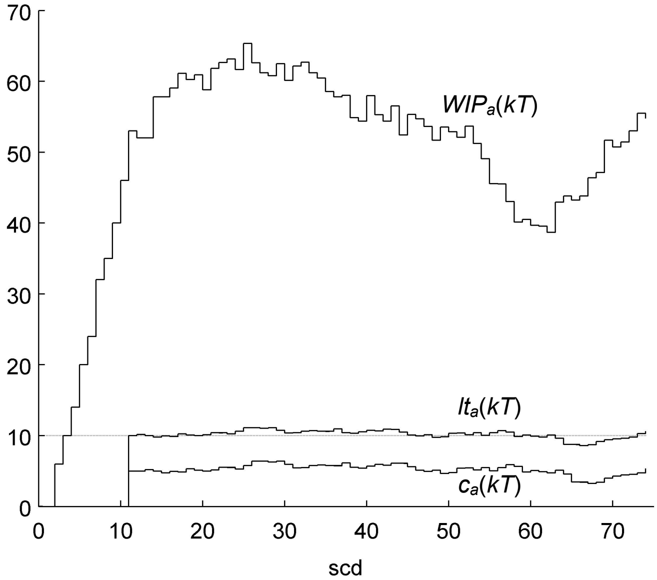

Figure 5: WIP [orders], lead time [scd], and production rate [orders/scd].

5 CONCLUSIONS

Control-theoretic analysis methods were used in this work to analyze the dynamic behavior of lead-time regulation in an autonomous work system. Two lead-time regulation topologies were studied: in the first, lead-time deviation, which is the difference between the desired lead time and the actual lead time, was used to adjust production rate; in the second, work deviation, which is the difference between work output and work input at the desired lead time earlier, was used to adjust production rate. Time-based simulations of lead-time regulation using these topologies yielded similar results, providing evidence that regulation using work deviation was a good approximation of regulation using lead-time deviation in this case. This is advantageous because the work-deviation topology permits analysis using control-theoretic methods, allowing selection of a constant value of its proportional control parameter that avoids both slow and oscillatory response. Implementation of work deviation regulation as a substitute for the perhaps more obvious lead-time deviation regulation may therefore be preferable when production rates vary significantly with time because its fundamental dynamic properties are not a function of production rate.

6 ACKNOWLEGEMENTS

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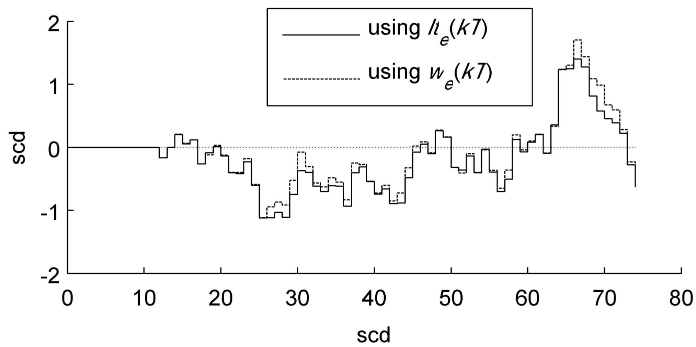

Figure 6: Lead-time deviation resulting from lead-time deviation topology and work deviation topology

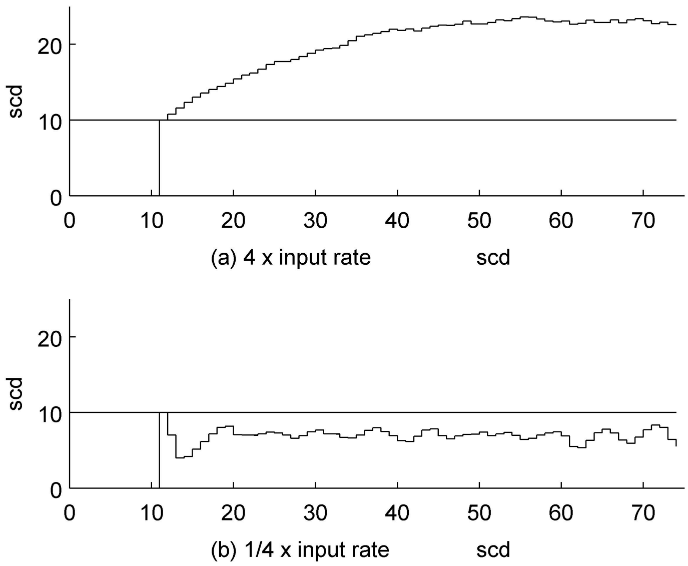

Figure 7: Lead-time deviation resulting from lead-time deviation topology without control parameter adjustment.