

Maintaining constant WIP-regulation dynamics in production networks with autonomous work systems

N.A. Duffie (1)^{a,*}, L. Shi^b

^a Department of Mechanical Engineering, University of Wisconsin-Madison, Madison, WI 53706, USA

^b Department of Industrial and Systems Engineering, University of Wisconsin-Madison, Madison, WI, USA

ARTICLE INFO

Keywords:
Production
Control
Analysis

ABSTRACT

In this paper, a method is presented for information sharing in production networks with large numbers of autonomous work systems for the purpose of maintaining constant dynamic properties when the structure of physical order flows between the work systems is omni-directional and variable. It is shown that information sharing is necessary if undesirable behaviors such as oscillation or slow response are to be avoided. A method for designing the dynamic properties of such networks is presented along with a method for distributed computation and communication of information needed to locally compensate for the expected order flows from other work systems.

© 2009 CIRP.

1. Introduction

The ability to establish and maintain desirable dynamic behavior is essential in production networks. This can be a particularly significant challenge when the individual work systems in a network have high levels of local autonomy, and cooperation and information sharing are used to ensure effective operation, rather than centralized control. Production networks are known to exhibit unfavorable dynamic behavior; for example, inventory levels can oscillate in supply chains as organizations respond individually to variations in orders [1]. Decentralized planning and control methods are an increasingly important alternative to centralized control of production networks; however, achieving effective cooperation and choosing the appropriate level of autonomy are significant challenges in design of these autonomous logistic systems [2–4].

Due to the complexity of interactions between decision-making entities in production networks, modeling their behavior also is a challenge [5,6]. Two-level models have been developed that combine control of Work In Progress (WIP) with control of backlog [7] and final inventory [8]. Application of control theory to the production inventory problem has been reviewed [9], and control-theoretic approaches have been used to model supply chain management including the use of differential equations to study the stability of adjustments in inventories and production rates [10]. Autonomous work systems require coupling structures that create the information-based interactions necessary to ensure that local actions are globally effective [11], and the control laws and heuristic rules chosen need to create well-behaved network dynamics including desired responsiveness, absence of oscillatory behavior, and robustness in the presence of uncertainties. There is a need to limit the propagation of disturbances in a production network and to ensure that the dynamic behavior of the network remains as designed and does not change unpredictably or unfavorably with time.

It is shown in this paper, through dynamic system analysis, that when the structure of order flows between the work systems is omni-directional and variable, there can be variations in the fundamental dynamic behavior of the work systems and the production network. It is also shown that information coupling created by sharing of order-flow structure information can produce desired and consistent dynamic behavior when the order-flow information is accurate. A method for designing the dynamic properties of a network is presented along with a method for distributed computation and communication of information needed to locally compensate for the expected order flows from other work systems.

2. Dynamic model

The WIP regulation topology for autonomous work systems shown in Fig. 1 was analyzed in which order-flow information is shared to anticipate and compensate for the expected dynamic effects of physical order flows between work systems. The work systems adjust capacity with the objective of maintaining a desired amount of local WIP, a logistic variable that is readily measured [12]. The desired WIP can be locally specified or planned at a higher level by entities outside the network, and it need not be constant. It is assumed that local capacity is periodically adjusted, daily or weekly for example. T is the time period between capacity adjustments (for example, one shop calendar day). WIP is assumed to be regulated using adjustments in full capacity that are delayed by time dT , d time periods, representing the realities of labor contracts and other logistic issues that prevent instantaneous adjustment of capacity. Fig. 2 shows the dynamic model of a network of N work systems. Time-domain definitions of the vectors in the model, elements of which are shown in Fig. 1, are as follows:

$\mathbf{i}(kT)$ actual rates at which orders are input to the work systems from sources external to the network;
 $\mathbf{w}_d(kT)$ work disturbances such as rush orders and order cancellations;

* Corresponding author.

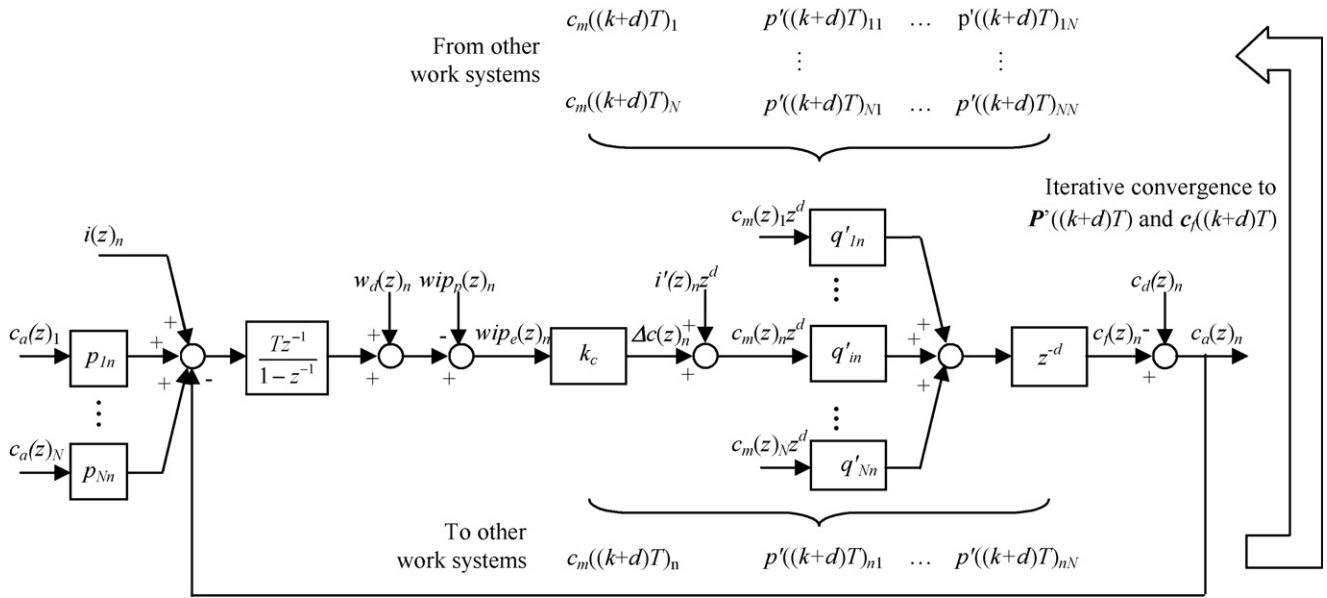


Fig. 1. WIP regulation in work system n , in which order-flow structure is cooperatively determined for the purpose of establishing and maintaining constant fundamental dynamic properties.

- $wip_p(kT)$ desired WIP;
- $i'(kT)$ expected input rates from sources external to the network;
- $c_f(kT)$ full capacity as determined by the capacity adjustment policy;
- $c_d(kT)$ capacity disturbances such as operator illness and equipment failure;
- $c_a(kT)$ actual capacity.

These represent continuous variables that are assumed to be constant over time $kT \leq t < (k + 1)T$ where $k = 0, 1, 2, \dots$. The total orders that have been input to and output from the work systems up to time kT are represented in the time domain by $w_i(kT)$ and $w_o(kT)$, respectively. Capacity limits, buffer size limitations, setup times, transportation times, variations in delay with capacity adjustment magnitude, etc. are not modeled. Orders are used as the dependent variable rather than hours of work content, with the assumption that orders are conserved as they move from work system to work system [13]. The units of work are orders and the units of capacity are orders per shop calendar day (orders/scd).

The following also are assumed to be constant over time $kT \leq t < (k + 1)T$: $\mathbf{P}^T(kT)$, a matrix in which each element $p^T(kT)_{nj}$ represents the expected fraction of the orders flowing out of work system n that flow into work system j ; $\mathbf{P}(kT)$, a matrix in which each element $p(kT)_{nj}$ represents the actual fraction of the orders flowing out of work system n that flow into work system j ; $\mathbf{P}_o(kT)$, a diagonal matrix in which element $p_o(kT)_{nm}$ represents the actual fraction of orders flowing out of work system n that flow out of the network. $\mathbf{P}(kT)$ and $\mathbf{P}_o(kT)$ represent the actual structure of order flow in the network. The information coupling between the work

systems is represented by

$$\mathbf{Q}^T(kT) = (\mathbf{I} - \mathbf{P}^T(kT))^{-1} \quad (1)$$

The transfer equations relating $wip_a(z)$ and $c_d(z)$ to the inputs $\mathbf{i}(z)$, $\mathbf{w}_d(z)$, $wip_p(z)$, $\mathbf{i}'(z)$ and $\mathbf{c}_d(z)$ when \mathbf{P}^T , \mathbf{P} and \mathbf{P}_o are constant:

$$wip_a(z) = ((1 - z^{-1})\mathbf{I} + k_c T(\mathbf{I} - \mathbf{P}^T)(\mathbf{I} - \mathbf{P}^T)^{-1} z^{-(d+1)})^{-1} (Tz^{-1}\mathbf{i}(z) + (1 - z^{-1})\mathbf{w}_d(z) + k_c T(\mathbf{I} - \mathbf{P}^T)(\mathbf{I} - \mathbf{P}^T)^{-1} z^{-(d+1)} wip_p(z) - T(\mathbf{I} - \mathbf{P}^T)(\mathbf{I} - \mathbf{P}^T)^{-1} z^{-d} \mathbf{i}'(z) + T(\mathbf{I} - \mathbf{P}^T)z^{-1} \mathbf{c}_d(z)) \quad (2)$$

$$c_a(z) = ((1 - z^{-1})\mathbf{I} + k_c T(\mathbf{I} - \mathbf{P}^T)(\mathbf{I} - \mathbf{P}^T)^{-1} z^{-(d+1)})^{-1} (k_c T(\mathbf{I} - \mathbf{P}^T)^{-1} \times z^{-(d+1)} \mathbf{i}(z) + k_c (\mathbf{I} - \mathbf{P}^T)^{-1} (1 - z^{-1}) z^{-d} \mathbf{w}_d(z) - k_c (\mathbf{I} - \mathbf{P}^T)^{-1} (1 - z^{-1}) z^{-d} wip_p(z) (\mathbf{I} - \mathbf{P}^T)^{-1} (1 - z^{-1}) \mathbf{i}'(z) - (1 - z^{-1}) \mathbf{c}_d(z)) \quad (3)$$

Fundamental dynamic properties of the network then are described by the roots of

$$\det((1 - z^{-1})\mathbf{I} + k_c T(\mathbf{I} - \mathbf{P}^T)(\mathbf{I} - \mathbf{P}^T)^{-1} z^{-(d+1)}) = 0 \quad (4)$$

3. Selection of control parameter value

In Figs. 1 and 2, a lower value of control parameter k_c tends to produce a more slow-acting dynamic system and, within limits, a higher value of k_c tends to produce a more fast-acting system. While each work system could have a different value of this control parameter, here it is assumed to be the same throughout the

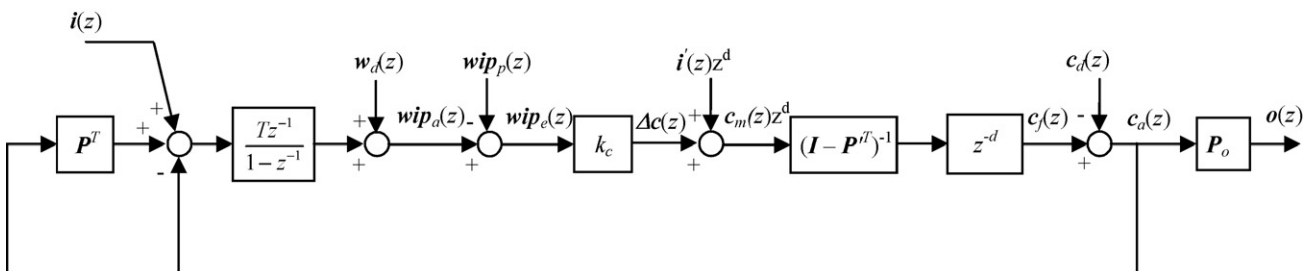


Fig. 2. WIP regulation in a network of autonomous work systems.

network. If the actual order-flow structure is approximately equal to the expected structure ($\mathbf{P}(kT) \approx \mathbf{P}'(kT)$), then it can be observed in Eq. (4) that the fundamental dynamic properties of the network are not a function of order flow structure and can be established by choosing k_c , given T and d . If it is desired to avoid slow or oscillatory response to turbulence, then

$$k_c = \frac{d^d}{(d+1)^{d+1}} \quad (5)$$

can be chosen, which results in two equal real roots, each associated with characteristic time:

$$\tau = \frac{-T}{\ln(d) - \ln(d+1)} \quad (6)$$

that tend to dominate the dynamic properties of both the individual work systems and the network.

4. Example with omni-directional order-flow structure

Consider an example in which one order type (120 orders out of a total of 659 orders) flows from shearing-sawing to quality control to heat treatment, creating the omni-directional order-flow structure shown in Fig. 3. If it is assumed that this order-flow structure does not vary with time, then

$$\mathbf{P} = \begin{bmatrix} 0 & 106/341 & 235/341 & 0 & 0 \\ 0 & 0 & 0 & 68/401 & 324/401 \\ 0 & 0 & 0 & 100/236 & 129/236 \\ 0 & 0 & 0 & 0 & 148/295 \\ 0 & 0 & 0 & 120/616 & 0 \end{bmatrix} \quad (7)$$

With $T = 1$ scd, $d = 1$, and $k_c = 0.25$ scd⁻¹ calculated using Eq. (5), the transfer functions for change in WIP in the heat treatment and quality control work systems as a function of work disturbances at the heat treatment work system, calculated using Eq. (2), are

$$\frac{\Delta w_i p_a(z)_4}{w_d(z)_4} = \frac{z(z-1)}{(z-0.5)^2} \quad (8)$$

$$\frac{\Delta w_i p_a(z)_5}{w_d(z)_4} = 0 \quad (9)$$

while the transfer functions without order-flow information sharing, calculated using Equation (2) with $\mathbf{P}' = 0$ are

$$\frac{\Delta w_i p_a(z)_4}{w_d(z)_4} = \frac{z(z-1)(z-0.5)^2}{(z-0.7796)(z-0.2204)(z^2-z+0.3282)} \quad (10)$$

$$\frac{\Delta w_i p_a(z)_5}{w_d(z)_4} = \frac{0.12542z(z-1)}{(z-0.7796)(z-0.2204)(z^2-z+0.3282)} \quad (11)$$

Table 1 shows the characteristic roots, damping ratios and characteristic times associated with these transfer functions, demonstrating that order-flow structure does not influence the system dynamics when there is order-flow information sharing. On the other hand, characteristic time of 4.0 scd indicates significantly less desirable behavior for the case without order-flow information sharing. Responses to a 1-order step work disturbance at the heat treatment work system with and without information sharing, calculated using Eqs. (8) through (11), are

Table 1
Dynamic characteristics of change in WIP in response to work disturbances in the heat treatment work system ($T = 1$ scd, $d = 1$, $k_c = 0.25$).

Work system	With order-flow information sharing		Without order-flow information sharing	
	Damping	Characteristic times [scd]	Damping	Characteristic times [scd]
Heat treatment	1.0	1.4	-	4.0
Quality control	-	-	-	0.7
	-	-	0.7	1.8

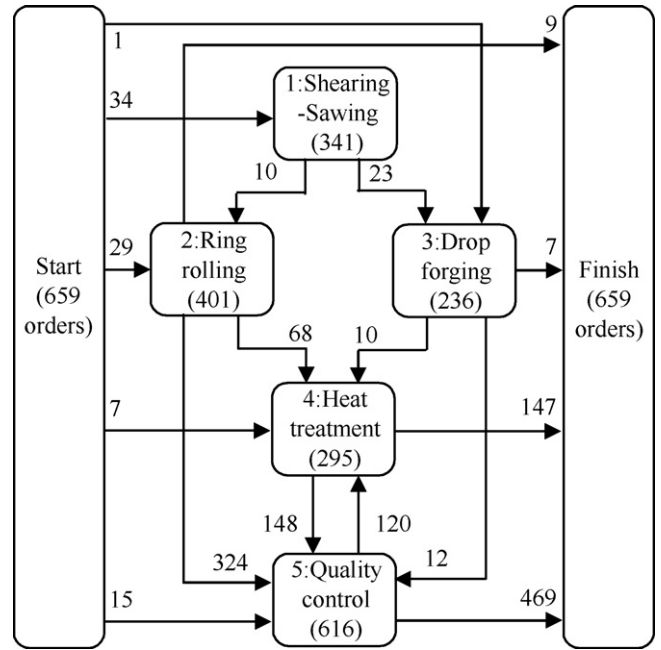


Fig. 3. Omni-directional order-flow structure example.

shown in Fig. 4, along with the responses without order-flow information sharing, calculated using Eqs. (2) with $\mathbf{P}' = 0$.

5. Cooperative determination of order-flow structure

If the order-flow structure within the network is time varying, then the work systems need to cooperatively determine the expected order-flow structure $\mathbf{P}'((k+d)T)$ time dT in advance. Each work system possesses its current planned WIP $w_{ip}(kT)_n$ and its expected input rates from sources external to the network $i((k+d)T)_n$, which is assumed to be known time dT in advance. If the order-flow structure during period $(k+d)T \leq t < (k+d+1)T$ is not significantly affected by full capacities $c_f((k+d)T)$, then the following algorithm can be used to adjust the capacity of work system n using shared order-flow information:

1. Work system n calculates $c_m((k+d)T)_n$ as indicated in Fig. 1. Also, using the orders (WIP) in its local queue, their next processing

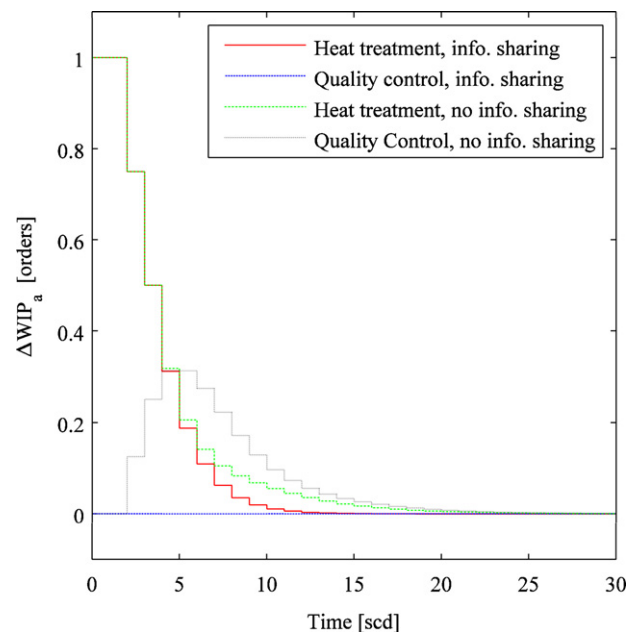


Fig. 4. Change in WIP in the heat treatment and quality control work systems in response to a 1-order work disturbance at the heat treatment work system.

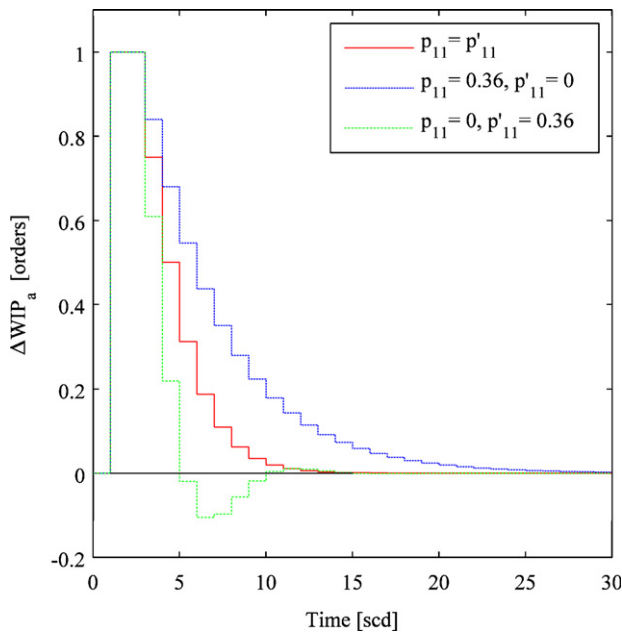


Fig. 5. Change in WIP in a single work system in response to a 1-order work disturbance for various expected and actual order-flow structures.

step, and previously calculated capacities $\mathbf{c}_f(kT)$, $\mathbf{c}_f((k+1)T)$, ..., $\mathbf{c}_f((k+d-1)T)$, the work system calculates expected flow fractions $\mathbf{p}'((k+d)T)_n = [p'((k+d)T)_{n1}, p'((k+d)T)_{n2}, \dots, p'((k+d)T)_{nN}]$.

2. Work system n sends $c_m((k+d)T)_n$ and $\mathbf{p}'((k+d)T)_n$ to all other work systems in the network.
3. Work system n assembles vector $\mathbf{c}_m((k+d)T)$ and matrix $\mathbf{P}'((k+d)T)$ from the information received from the other work systems, and then calculates $\mathbf{Q}'((k+d)T)$ using Eq. (1).
4. Work system n calculates its full capacity time dT in advance using

$$c_f((k+d)T)_n = \sum_{j=1}^N q'((k+d)T)_{jn} c_m((k+d)T)_j \quad (12)$$

However, if the order-flow structure changes significantly as a function of $\mathbf{c}_f((k+d)T)$, for example when few orders are produced in period T and their routings differ significantly, then this algorithm is executed iteratively as indicated in Fig. 1 until the capacities and order-flow structure converge to final values. The rates of change of these variables may need to be attenuated during this iterative process to ensure convergence. The amount of data that needs to be communicated between work systems and computations required are relatively small and convergence can be reached in a period that is insignificant compared to T .

6. Single work system with reentry order flow

The importance of obtaining good estimates of the order-flow structure can be illustrated by considering a single work system in which a fraction p_{11} of its output order flow reenters the work system. The single work system is assumed to possess an estimate p'_{11} of p_{11} ; hence, p'_{11} represents the expected order-flow structure and p_{11} represents the actual structure. The change in WIP in response to a 1-order work disturbance (for example, an unplanned rush order), obtained from Eq. (2) is shown in Fig. 5 where it can be seen that when the actual order-flow structure differs from the expected structure, dynamic properties change significantly. A 36% reentry order flow when 0% is expected significantly increases the time required to recover from a work disturbance, while a 0% reentry flow when 36% is expected results in oscillatory behavior.

7. Conclusions

It has been shown through control-theoretic dynamic analysis that a WIP regulation topology for autonomous work systems that

includes order-flow information sharing can lead to more favorable and more consistent fundamental dynamic behavior. The goal of this dynamic consistency is to allow the responsiveness of regulation to be chosen (designed) and relied upon in network operation. There is a high level of autonomy because only local information is shared between work systems. Information is gathered and shared when it is needed rather than being archived either within the work systems or in a centralized network database.

It has been shown in this paper that variations in the coupling between work systems created by omni-directional physical order flows can be result in variation in the fundamental dynamic behavior of the work systems and the production network. It is also shown that information coupling created by sharing of order-flow structure information can produce desired and consistent dynamic behavior when the order-flow information is accurate. As illustrated in Fig. 4, order-flow information sharing curtails propagation of turbulence to downstream work systems. On the other hand, as illustrated in Fig. 5, and inaccurate compensation for order-flow structure can result in dynamic behavior that deviates from that desired, potentially becoming oscillatory or requiring longer time periods to react to disturbances than designed.

A method was presented for choosing a value of control parameter k_c that improves responsiveness without producing oscillation, and an algorithm has been presented for distributed computation and communication of information needed to locally compensate for the expected order flows from other work systems. Delay in capacity adjustment has been included to represent the inability to make instantaneous adjustments, but non-linear variations in cost, delay and feasibility with capacity adjustment magnitude have not been modeled. Other logistic issues have not been addressed including the effects of capacity limits, buffer capacities, setup times, transportation times, starvation of work systems when WIP is low, and modeling the work content of orders.

Acknowledgements

This work was supported in part by the U.S. National Science Foundation under grant DMI-0646697 and the German Research Foundation under grants SFB 637/2-A6 and Br 933/16-1.

References

- [1] Huang GQ, Lau JSK, Mak KL (2003) The Impacts of Sharing Production Information on Supply Chain Dynamics: A Review of the Literature. *International Journal Of Production Research* 41(7):1483–1517.
- [2] Prabhu VV, Duffie NA (1995) Modeling and Analysis of Non-linear Dynamics in Autonomous Heterarchical Manufacturing Systems Control. *Annals of the CIRP* 44(1):425–428.
- [3] Böse F, Windt K (2007) Catalogue of Criteria for Autonomous Control in Logistics. in Hülsmann M, Windt K, (Eds.) *Understanding Autonomous Cooperation and Control in Logistics*. Springer-Verlag, Berlin, Heidelberg, pp. 57–72.
- [4] Duffie N (2007) Dynamics in Logistics. in Haasis H-DKreowski H-JScholz-Reiter B, (Eds.) *Proceedings of the 1st International Conference on LDIC 2007*:3–24.
- [5] Wiendahl H-P, Lutz S (2002) Production in Networks. *Annals of the CIRP* 51(2):573–586.
- [6] Scholz-Reiter B, Freitag M, Schmieder F (2002) Modeling and Control of Production Systems Based on Nonlinear Dynamics Theory. *Annals of the CIRP* 51(1):375–378.
- [7] Kim J-H, Duffie N (2004) Backlog Control Design for Closed-loop PPC Systems. *Annals of the CIRP* 53(1):357–360.
- [8] Deif AM, El Maraghy WH (2006) Effect of Time-based Parameters on the Agility of a Dynamic MPC System. *Annals of the CIRP* 55(1):437–440.
- [9] Ortega M, Lin L (2004) Control Theory Applications to the Production-inventory Problem: A Review. *International Journal Of Production Research* 42(11):2303–2322.
- [10] Diseny SM, Towill DR (2002) A Discrete Transfer Function Model to Determine the Dynamic Stability of a Vendor Managed Inventory Supply Chain. *International Journal Of Production Research* 40(1):179–204.
- [11] Kim J-H, Duffie N (2006) Performance of Coupled Closed-loop Workstation Capacity Controls in a Multi-workstation Production System. *Annals of the CIRP* 55(1):449–452.
- [12] Nyhuis P (2006) Logistic Production Operating Curves—Basic Model of the Theory of Logistic Operating Curves. *Annals of the CIRP* 55(1):441–444.
- [13] Schneider M, Wiendahl H-P (2003) *Logistic Measurement of Manufacturing Departments by the Use of Logistic Process Operating Curves (LPOCs)*. Institute of Production Systems and Logistics (IFA), University of Hanover, Germany.