

Dynamic modeling of production networks of autonomous work systems with local capacity control

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ABSTRACT

In this paper, a dynamic model is presented for production networks with a potentially large number of autonomous work systems, each having local capacity control. The model allows fundamental dynamic properties to be predicted using control-theoretic methods, together with the response of variables such as work-in-progress and lead-time for the network and its individual work systems. This is illustrated using industrial data. The behavior of one of the work systems in this network is analyzed further, and the results are compared with results obtained using a discrete event simulation model.

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1. Introduction

Production networks are emerging as a new type of cooperation between and within companies, requiring new techniques and methods for their operation and management [1]. Coordination of resource use is a key challenge in achieving short delivery times and delivery time reliability. These networks can exhibit unfavorable dynamic behavior as individual organizations respond to variations in orders in the absence of sufficient communication and collaboration, leading to recommendations that supply chains should be globally rather than locally controlled and that information sharing should be extensive [2,3]. However, the dynamic and structural complexity of these emerging networks inhibits collection of the information necessary for centralized planning and control, and decentralized coordination must be provided by logistic processes with autonomous capabilities [4]. Dynamic models will be an important tool in understanding the behavior of networks with decentralized control and enabling design of effective autonomous logistic processes.

A production network with several autonomous work systems is depicted in Fig. 1. The behavior of such a network is affected by external and internal order flows, planning, internal disturbances, and the control laws used locally in the work systems to adjust resources for processing orders. In prior work, sharing of capacity information between work systems has been modeled [5] along with the benefits of alternative control laws and reducing delay in capacity changes [6]. Several authors have described both linear and nonlinear dynamical models for control of variables such as inventory levels and work in progress (WIP), including the use of pipeline flow concepts to represent lead times and production delays [7]. A closed-loop production planning and control concept has been employed with adaptive inventory control in decision support systems in a multi-product

medical supplies market [8]. State-space models have been used for switching between a library of optimal controllers to adjust WIP in serial production systems in the presence of machine failures [9], and switching of control policies in response to market strategies has been investigated [10]. In this paper, the focus is on development of a discrete state-space dynamic model for production networks with an arbitrarily large number of work systems, illustrating the use of this generic model to predict performance, and comparing the results with results obtained using discrete event simulation.

2. Model of multiple work systems

In Fig. 1, the elements of vector $\mathbf{i}(nT)$ represent the rate at which orders flow from external sources into N work systems in a network at discrete instants in time separated by time interval T (e.g. 1 shop calendar day [scd]), where $n = 0, 1, 2, \dots$. The total number of orders that have been input to each work system then can be represented by the elements of vector

$$\mathbf{w}_i((n+1)T) = \mathbf{w}_i(nT) + T(\mathbf{i}(nT) + \mathbf{P}^T \mathbf{c}_a(nT)) \quad (1)$$

where $\mathbf{c}_a(nT)$ is the actual capacity (rate at which orders are output) of each work system and \mathbf{P} is a matrix in which element p_{jk} approximates the fraction of the flow out of work system j that flows into work system k [11]. The total number of orders that have been output by each work system is

$$\mathbf{w}_o((n+1)T) = \mathbf{w}_o(nT) + T\mathbf{c}_a(nT) \quad (2)$$

and the WIP is

$$\mathbf{wip}_a(nT) = \mathbf{w}_i(nT) - \mathbf{w}_o(nT) + \mathbf{w}_d(nT) \quad (3)$$

where $\mathbf{w}_d(nT)$ represents local work disturbances, such as rush orders, that affect the work systems.

If it is desired to maintain the WIP in each work system in the vicinity of planned levels $\mathbf{wip}_p(nT)$, then the capacities of the work

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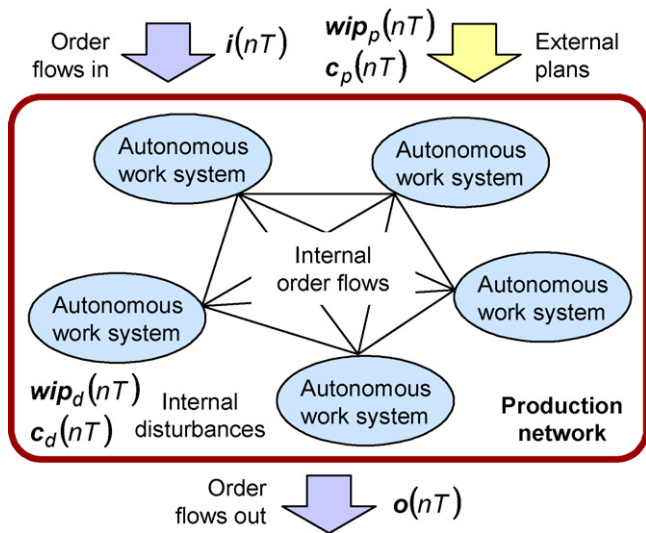


Fig. 1. Production network consisting of a group of autonomous work systems.

systems can be adjusted with respect to their planned capacities $c_p(nT)$ using straightforward proportionality k_c :

$$c_m(nT) = k_c(\mathbf{wip}_a(nT) - \mathbf{wip}_p(nT)) \quad (4)$$

$$c_a(nT) = c_p(nT) + c_m((n-d)T) - c_d(nT) \quad (5)$$

where $c_d(nT)$ represents local capacity disturbances such as equipment failures. The value of k_c determines how quickly differences between actual and planned WIP are eliminated, and the whether work system responses are fundamentally oscillatory (regardless of whether there are oscillations in their inputs). Furthermore, capacity changes were assumed to be delayed by a time period dT , but planned capacity and WIP were assumed to be known in advance and did not require delayed implementation. Although factors such as setup times and WIP are known to affect performance [12], it was assumed that the actual capacity of the work systems was equal to their full capacity of the work systems. There was no information sharing between work systems.

Eqs. (1) through (5) can be combined and transformed to obtain a discrete model for the system:

$$\begin{bmatrix} z\mathbf{w}_i(z) \\ z\mathbf{w}_o(z) \\ z^d\mathbf{c}_m(z) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & T\mathbf{P}^T \\ \mathbf{0} & \mathbf{I} & T\mathbf{I} \\ k_c\mathbf{I} & -k_c\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w}_i(z) \\ \mathbf{w}_o(z) \\ \mathbf{c}_m(z) \end{bmatrix} + \begin{bmatrix} T\mathbf{I} & \mathbf{0} & \mathbf{0} & T\mathbf{P}^T & -T\mathbf{P}^T \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & T\mathbf{I} & -T\mathbf{I} \\ \mathbf{0} & k_c\mathbf{I} & -k_c\mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}(z) \\ \mathbf{w}_d(z) \\ \mathbf{wip}_p(z) \\ \mathbf{c}_p(z) \\ \mathbf{c}_d(z) \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \mathbf{w}_i(z) \\ \mathbf{w}_o(z) \\ \mathbf{c}_a(z) \\ \mathbf{wip}_a(z) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w}_i(z) \\ \mathbf{w}_o(z) \\ \mathbf{c}_m(z) \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{i}(z) \\ \mathbf{w}_d(z) \\ \mathbf{wip}_p(z) \\ \mathbf{c}_p(z) \\ \mathbf{c}_d(z) \end{bmatrix} \quad (7)$$

It is desirable to obtain a discrete state-space-model that is compatible with dynamic systems analysis software (Matlab was used to obtain the results in this paper); therefore, if $d = 0$ then the state $\mathbf{c}_m(z)$ should be eliminated in Eq. (6), whereas Eq. (6) should be augmented if $d > 1$ so that there are d states $\mathbf{c}_m(z)$, $\mathbf{c}_m'(z)$, $\mathbf{c}_m''(z)$, etc., each representing a delay of T in capacity adjustment. As a result, there will be $N(d+2)$ states, $5N$ inputs

and $4N$ outputs in the dynamic model for the production network. Hence, its dimensions grow linearly with the number of work systems and linearly with the delay in capacity adjustment. Two outputs can be added to Eq. (7), if desired, that represent the total orders input to the network and output from the network:

$$\mathbf{w}_{in}(nT) = [1 \ 1 \ \dots \ 1](\mathbf{w}_i(nT) - \mathbf{P}^T\mathbf{w}_o(nT)) \quad (8)$$

$$\mathbf{w}_{out}(nT) = [1 \ 1 \ \dots \ 1]\mathbf{P}_0\mathbf{w}_o(nT) \quad (9)$$

where \mathbf{P}_0 is the diagonal matrix with non-zero elements

$$p_{0_{jj}} = 1 - \sum_{k=1}^N p_{jk} \quad (10)$$

Furthermore, the rate at which orders flow out of the network from the individual work systems is

$$\mathbf{o}(nT) = \mathbf{P}_0\mathbf{c}_a(nT) \quad (11)$$

3. Multiple work system example

Application of this discrete state-space dynamic model was illustrated using data from a forging company that supplies components to the automotive industry. The company's basic product is starter ring gears; other products include sensor wheels with machined teeth and flywheel assemblies for manual transmissions. The data documents 659 orders that entered the system from scd 162 to scd 347 in the year 2001. For purposes of analysis, the production system was grouped into the five work systems listed in Table 1. Eqs. (6) and (7) accommodate planned capacities and WIP that vary with time. The planned capacities and WIP listed in Table 1 they are averages obtained from the data. The time periods over which the plan was used in the model are also listed in Table 1. The planned capacities and WIP were zero 14 days before and after these periods; however, Eqs. (4) and (5) allow output rates and WIP to be greater than zero at such times. The internal flow of orders was approximated using the following matrix, in which element p_{ij} is the total number of orders that went from work system i to work system j divided by the total number of orders that left work system i :

$$\mathbf{P} = \begin{bmatrix} 0 & 106/341 & 235/341 & 0 & 0 \\ 0 & 0 & 0 & 188/401 & 204/401 \\ 0 & 0 & 0 & 100/236 & 129/236 \\ 0 & 0 & 0 & 0 & 268/295 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

Fig. 2 shows the order flows into each work system in the network. With these inputs, the plan in Table 1, the order flow matrix in Eq. (12), and zero initial conditions, the dynamic mode with $k_c = 0.25 \text{ scd}^{-1}$ and $T = 1 \text{ scd}$ predicts the throughput shown in Fig. 3.

A transfer function can be obtained for each output/input combination in Eq. (7); for example, the change in actual capacity that results from capacity disturbances in the Shearing/Sawing

Table 1
Planned capacity and WIP

Work system	c_p (orders/scd)			wip_p (orders)	Duration ^a (scd)
	Monday–Friday	Saturday	Sunday		
1. Shear/Sawing	4.72	0.92	0.00	21.07	181–244
2. Ring rolling	5.34	1.50	0.00	18.92	181–244
3. Drop forging	2.95	0.42	0.00	14.46	181–244
4. Heat treat.	2.70	2.50	1.92	14.87	181–244
5. Quality control	6.28	0.83	0.08	72.11	188–265

^a Preceded/followed by a 14-day linear ramp up/down period.

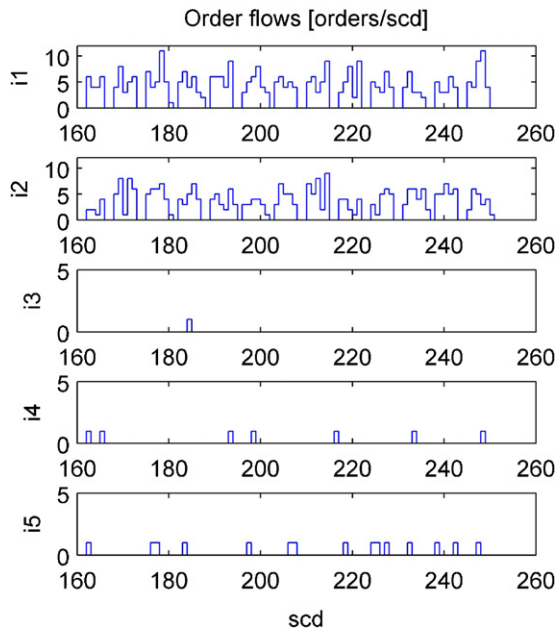


Fig. 2. External order flows into work systems.

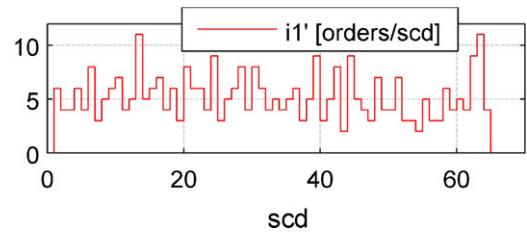


Fig. 4. Input to Shearing/Sawing workstation with weekends eliminated.

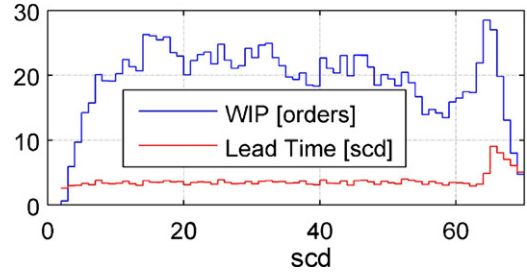


Fig. 5. WIP and lead time from dynamic model.

work system are

$$\frac{\Delta c_{a1}(z)}{c_{d1}(z)} = \frac{-z(z-1)}{(z-0.5)^2} \quad (13)$$

$$\frac{\Delta c_{a2}(z)}{c_{d1}(z)} = \frac{-0.7771z(z-1)}{(z-0.5)^4} \quad (14)$$

$$\frac{\Delta c_{a3}(z)}{c_{d1}(z)} = \frac{-0.1723z(z-1)}{(z-0.5)^4} \quad (15)$$

$$\frac{\Delta c_{a4}(z)}{c_{d1}(z)} = \frac{-0.0274z(z-1)}{(z-0.5)^6} \quad (16)$$

$$\frac{\Delta c_{a5}(z)}{c_{d1}(z)} = \frac{-0.0331z(z-1)(z^2-z+0.4375)}{(z-0.5)^8} \quad (17)$$

Peak changes in output rate of 1, 0.078, 0.172, 0.062 and 0.077 orders/scd in the five work systems, respectively, and response times (98% final) of 13, 17, 17, 24 and 28 scd, respectively, are predicted responses to an unplanned capacity reduction in the Shearing/Sawing work system of 1 order/scd for 1 scd.

4. Dynamic analysis of single work system

The Shearing/Sawing work system did not receive input from other work systems, and it was selected for further analysis to assess the dynamic behavior of WIP and lead time in a work system with autonomous capacity adjustment. To more clearly illustrate behavior, the input data were modified to eliminate weekends, with the result shown in Fig. 4. With

Table 2
Behavior the Shearing/Sawing work system

	WIP (orders)		Lead time (scd)	
	Mean	σ	Mean	σ
Dynamic model	21.03	3.36	3.58	0.80
Discrete-event simulation	19.30	3.16	3.53	0.53

planned capacity and WIP constant at 5.32 orders/scd and 21.07 orders, respectively, the WIP and lead time predicted by the dynamic model are shown in Fig. 5. Table 2 shows the mean and standard deviation for WIP and lead time, calculated from scd 10 to 65 to eliminate startup and shutdown trends.

A discrete event simulation (DES) model of the Shearing/Sawing work system was constructed using Arena. As indicated in Fig. 6, there were two modules in the model: a single-station input-process-output module; and a WIP control module. Order arrival was modeled as a Poisson process, driven by the industrial data in Fig. 4. Three identical work stations were fed from a single queue. The WIP control logic was executed at 8:00 in the morning, calculating the deviation between the current WIP (the sum of orders in queue and the orders being serviced) and the planned WIP, and then adjusted the service time of the incoming jobs using Eqs. (4) and (5). Planned capacity evolved during the simulation, as the total orders processed divided by total time, and the maximum service time was 1 scd. Table 2 also shows the mean and standard deviation for WIP and lead time obtained from the DES model. For this example, the results are similar in spite of the Poisson arrival process, detailed queue modeling, and continuous time-resolution of the DES model.

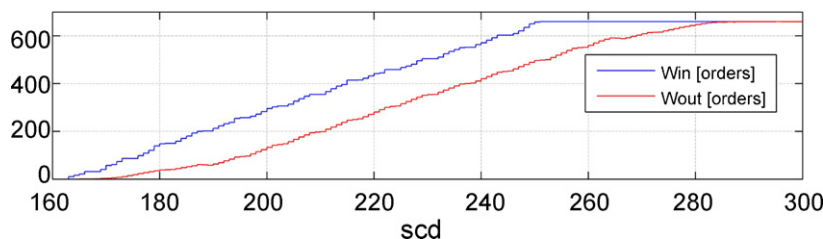


Fig. 3. Total orders into and out of the production network.

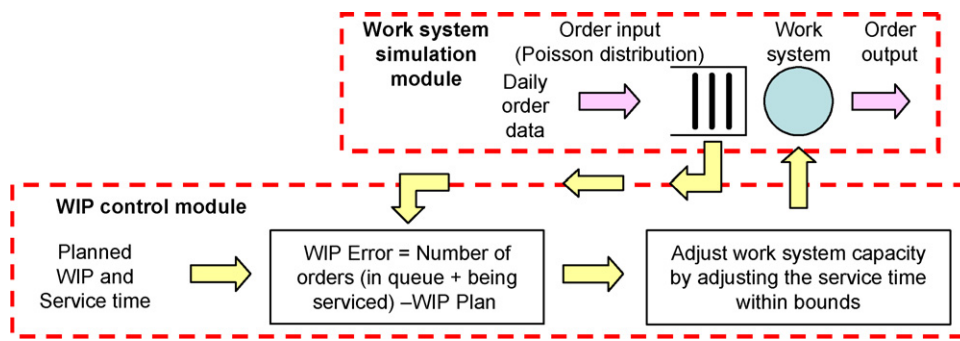


Fig. 6. Discrete-event simulation model of Shearing/Sawing work system.

5. Conclusions

The dynamic model that has been presented represents the fundamental behavior of a production network with an arbitrarily large number of autonomous work systems with local capacity control. It has been shown that transfer functions are readily obtained that represent the inter- and intra-work-system relationships between the outputs (orders input, orders processed, order output rates and WIP) and inputs (external input, work disturbances, planned WIP and capacity disturbances) of all work systems. In the production network example that was studied, WIP remained in the vicinity of the planned value, with variations due to fluctuations in external order inputs. Furthermore, lead times were held relatively constant with a simple proportional WIP-control law even with no exchange of information between work systems.

One of the work systems in the network was analyzed further, and its response was characterized using results from both the dynamic model and a DES model, which also incorporated local capacity adjustment. Similar means and variations were predicted for WIP and lead time; however, greater differences are expected in future comparisons as DES models become more detailed and large numbers of work systems are incorporated. Tradeoffs need to be better understood between the complexity of detailed DES models that accurately model response in given scenarios and the potentially lower fidelity of control-theoretic dynamic models that can characterize fundamental dynamic properties. Supporting intuitive trial-and-error methodologies and theoretical methodologies, respectively, these approaches will be complementary tools in the design of autonomous logistic processes for production networks.

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References

- [1] Wiendahl H-P, Lutz S (2002) Production in Networks. *Annals of the CIRP* 52(2):573–586.
- [2] Helo P (2000) Dynamic Modelling of Surge Effect and Capacity Limitation in Supply Chains. *International Journal of Production Research* 38(17):4521–4533.
- [3] Huang GQ, Lau JSK, Mak KL (2003) The Impacts of Sharing Production Information on Supply Chain Dynamics: A Review of the Literature. *International Journal of Production Research* 41(7):1483–1517.
- [4] Scholz-Reiter B, Windt K, Freitag M (2004) Autonomous Logistic Processes: New Demands and First Approaches. *Proceedings of 37th CIRP International Seminar on Manufacturing Systems*, Budapest, Hungary, May 19–21, 357–362.
- [5] Kim J-H, Duffie N (2006) Performance of Coupled Closed-Loop Workstation Capacity Controls in a Multi-Workstation Production System. *Annals of the CIRP* 55(1):449–452.
- [6] Kim J-H, Duffie N (2005) Design and Analysis of Closed-Loop Capacity Control for a Multi-Workstation Production System. *Annals of the CIRP* 54(1):455–458.
- [7] John S, Naim MM, Towill DR (1994) Dynamic Analysis of a WIP Compensated Decision Support System. *International Journal of Manufacturing System Design* 1(4):283–297.
- [8] Towill DR, Evans GN, Cheema P (1997) Analysis and Design of an Adaptive Minimum Reasonable Inventory Control System. *Journal of Production Planning and Control* 8(6):545–577.
- [9] Ma Y-H, Koren Y (2004) Operation of Manufacturing Systems with Work in Process Inventory and Production Control. *Annals of the CIRP* 53(1):361–365.
- [10] Deif AM, ElMaraghy WH (2006) Effect of Time-Based Parameters on the Agility of a Dynamic MPC System. *Annals of the CIRP* 55(1):437–440.
- [11] Wiendahl H-P, Breithaupt J-W (2000) Automatic Production Control applying Control Theory. *International Journal of Production Economics* 63(1):33–46.
- [12] Nyhuis P, Cieminski G, Fischer A (2005) Applying Simulation and Analytical Models for Logistic Performance Prediction. *Annals of the CIRP* 54(1):417–422.