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Autonomous control methods in logistics – A mathematical perspective

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ABSTRACT

We model general autonomously controlled production networks by means of nonlinear differential equations and implement autonomous control methods, where transportation times and disturbances in the transportation times are taken into account. Autonomous control enables intelligent logistic objects to route themselves through a logistic network. Based on this model we investigate a certain scenario of a production network, where we show advantages and disadvantages of the implementation of autonomous control methods from a mathematical perspective in view of robustness and stability.

1. Introduction

Production systems, supply networks and other logistic structures are typical examples of complex systems with a nonlinear behavior. Their dynamics is subject to many different perturbations due to changes on market, changes in customer behavior, information and transport congestions, unreliable elements of the network etc. One of the approaches to handle such complex systems is to shift from centralized to decentralized or autonomous control, i.e., to allow the plants of a network to make their own decisions based on some given rules and available local information.

The term production network is used to describe company or cross-company owned networks with geographically dispersed plants. The primary objective of production networks is to achieve economies of scale through joint planning of production processes, a mutual use of common resources and integrated planning value added processes [1].

The main idea of autonomous cooperating logistic processes is to enable intelligent logistic objects to route themselves through a logistic network according to their own objectives and to make and execute decisions, based on local information [2]. In this context, intelligent logistic objects may be physical or material objects, e.g., parts or machines, as well as immaterial objects (e.g., production orders, information). By an engineering point of view it has been already shown that different autonomous control methods can help to increase the logistics performance and robustness of single production systems [3,4]. Due to the high structural and dynamical complexity of production networks one may expect that autonomous control has a positive effect on the dynamical behavior of these networks. This was confirmed by investigations of the performance of autonomously controlled production networks from an engineering point of view [5].

The queue length estimator (QLE) is an autonomous control method, which enables logistic objects in a production system to estimate the waiting and processing times of different alternative processing resources. It uses exclusively local

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information to evaluate the states of the alternatives. The application of this method leads to a better systems performance regarding throughput times compared to classical scheduling algorithms in highly dynamic situations [4,6].

The pheromone based autonomous control method (PHE) is a bio-inspired strategy. This approach is based on the idea to imitate the process of ants, marking possible routes to food sources. On the way between the nest and food sources ants emit pheromones such that other ants can detect those pheromones. They follow the trail with the highest concentration of pheromones [7]. This is transferred to production networks: On their route through the network, parts leave information about their processing time at a corresponding plant. Following parts which have to execute a decision at a network stage compare this artificial pheromone concentration and choose the plant with the highest concentration. Thus, the PHE depends on processing and waiting times.

In this paper we analyze the application of different autonomous control methods in production networks and display the advantages and disadvantages in view of robustness and stability from the mathematical point of view.

Therefore, we model general production networks by differential equations. Taking into account transportation times and disturbances during the transportation process, such as traffic jams or breakdowns of vehicles, for example, we use functional differential equations for the modeling. The QLE and PHE are described mathematically and implemented in the production network model.

The modeling is demonstrated in a certain scenario of a production network, where we apply autonomous control methods. We discuss the advantages and disadvantages of implementing autonomous control in contrast to a central planning from the mathematical point of view. At first, we take into account transportation times and then breakdowns of plants. We compare the system states for this case using the modeling with a central planning and the implementation of autonomous control methods. Then, we derive conditions to guarantee stability of a network using the different control methods under consideration of disturbances. Analyzing these conditions, we give conclusions about the advantages and disadvantages of the different control methods in view of stability.

Roughly speaking, for production networks stability means that the state of the network remains bounded over time, while all external inputs are bounded. Typical examples of unstable behavior are unbounded growth of unsatisfied orders or unbounded growth of the queue of the workload of a plant or a machine. This causes high inventory costs and loss of customers. To avoid instability of a network it is worth to investigate its behavior in advance. In particular mathematical modeling and analysis provide helpful tools for design, optimization and control of such networks and for deeper understanding of their dynamical properties. Stability analysis of production networks were performed in [8–11] for example. In [9] a scheme to identify parameter constellations which guarantee stability was presented.

In this contribution we identify the state as the number of unprocessed parts, which is the sum of the queue length and the WIP. Thus, stable behavior of the network is decisive for the performance and vitality of a network.

The structure of the paper is as follows. In Section 2 we model general production networks and both autonomous control methods. Based on this, we analyze a certain scenario of a production network in Section 3 implementing the QLE and PHE and considering plant breakdowns, where one advantage using autonomous control methods in contrast to a central planning is shown. In Section 4 we perform a stability analysis of the certain scenario and discuss advantages/disadvantages of the implementation of autonomous control methods in Section 5. Finally, the conclusions and some approaches for future research topics are included in Section 6.

2. Modeling

In this section we model general production networks and implement the QLE and PHE.

We consider production networks, consisting of n plants. Each plant is called a subsystem of the production network, which we call the whole network. For simplicity we assume, that there is only one unified type of material, i.e., all primary products, used in the production network, can be measured as a number of units of this unified material.

The state $x_i(t)$ of the ith subsystem at time $t \in \mathbb{R}_+$ is the quantity of unprocessed material within the ith subsystem at time t. The state of the whole network is denoted by $x(t) = (x_1(t), \ldots, x_n(t))^T \in \mathbb{R}^n$, where x^T denotes the transposition of a vector. A subsystem can get material from an external source, which is denoted by $u_i(t)$, and from subsystems of the network (internal inputs).

2.1. General networks and autonomous control without time-delays

At first, we consider production networks without transportation times and use ordinary differential equations (ODEs) for the modeling. ODEs describe the evolution of the state of the system with continuous time $t \in \mathbb{R}_+$, where $\mathbb{R}_+ := [0, \infty)$. The ith subsystem processes the raw material from its inventory with the rate $\tilde{f}_{ii}(x(t)) \geqslant 0$ and sends the produced goods to the jth subsystem with the rate $\tilde{f}_{ji}(x(t))$ or to some customers that are not considered in the network.

We will investigate the special case $\tilde{f}_{ji}(x(t)) = c_{ji}(x(t))\tilde{f}_i(x_i(t)), c_{ji} \in \mathbb{R}_+$ and $\tilde{f}_{ii}(x(t)) = \tilde{c}_{ii}(x(t))\tilde{f}_i(x_i(t)), \tilde{c}_{ii} \in \mathbb{R}_+$, where $\tilde{f}_i(x_i(t))$ is a continuous, strictly increasing function with $\tilde{f}_i(0) = 0$ and it is proportional to the processing rate of the system. $0 \leqslant c_{ji}(x) \leqslant 1$, $i \neq j$ are some positive distribution coefficients. We interpret the constant distribution coefficients as prescribed by a central planning and on the other hand variable distribution coefficients can be used for implementing some autonomous control method.

Under these assumptions the dynamics of the ith subsystem is described by an ODE of the form

$$\dot{x}_i(t) = \sum_{j=1, j\neq i}^{n} c_{ij}(x(t))\tilde{f}_j(x_j(t)) + u_i(t) - \tilde{c}_{ii}(x(t))\tilde{f}_i(x_i(t)), \quad i = 1, \ldots, n, \tag{1}$$

where the inflow of the ith subsystem is the input of material from other subsystems of the network, described by the term $\sum_{j=1,j\neq i}^{n} c_{ij}(x(t))\tilde{f}_j(x_j(t))$, and from an external source u_i. The outflow of the subsystem is the processing rate $\tilde{c}_{ii}(x(t))\tilde{f}_i(x_i(t))$. Denoting $c_{ii} := -\tilde{c}_{ii}$ the whole system can be written as

$$\dot{x}(t) = C(x(t))\tilde{f}(x(t)) + u(t), \tag{2}$$

where $x = (x_1, \ldots, x_n)^T, \tilde{f}(x(t)) = (\tilde{f}_1(x_1(t)), \ldots, \tilde{f}_n(x_n(t)))^T$, $u(t) = (u_1(t), \ldots, u_n(t))^T$ and $C \in \mathbb{R}^{n\times n}$, $C = (c_{ij})_{n\times n}$. Note that the conditions $\tilde{f}_i \in \mathscr{K}_\infty, c_{ii} < 0$ and $c_{ij} \geqslant 0$, $i \neq j$ imply, that if $x(0) \geqslant 0$ (that is $x_i(0) \geqslant 0 \forall\ i = 1, \ldots, n$), then $x(t) \geqslant 0$ for all $t > 0$.

As mentioned before, by the distribution rates c_{ij}, $i \neq j$, we implement different control methods, where constant choices represent the central planning. For the QLE we choose $c_{ij}(x(t)) = c_{ij}^q(x(t))$ as

$$c_{ij}^q(x(t)) := \frac{\frac{1}{x_i(t)+\varepsilon}}{\sum_k \frac{1}{x_k(t)+\varepsilon}},$$

where the index k denotes all subsystems which get material from subsystem j and $\varepsilon > 0$, arbitrarily small, is inserted to let the fraction be well-defined. The interpretation of c_{ij}^q is in simple words the following: if the queue length of the ith subsystem is small, then more material will be send to subsystem i in contrast to the case where x_i is large and c_{ij}^q is small. The method is based on the queue lengths of the subsystems.

The PHE is implemented by $c_{ij}(x(t)) = c_{ij}^p(x(t))$ and

$$c_{ij}^p(x(t)) := (1 - \upsilon_i)\frac{\tilde{f}_i(x_i(t))}{\sum_k \tilde{f}_k(x_k(t)) + \varepsilon} + \sum_{k\neq i} \upsilon_k \frac{\tilde{f}_k(x_k(t))}{\sum_q \tilde{f}_q(x_q(t)) + \varepsilon},$$

where k, q are indices denoting the subsystems which get material from subsystem j, $\varepsilon > 0$ and $0 \leqslant \upsilon_i \leqslant 1$ is the evaporation constant of the ith subsystem. In contrast to the QLE, this choice takes into account the actual processing rates. By the evaporation constant υ_i one can justify the PHE method in order to increase the performance or robustness of the network. In the following sections we will investigate this constant more detailed.

Remark 2.1. It holds $0 \leqslant c_{ij}^q \leqslant 1, 0 \leqslant c_{ij}^p \leqslant 1$. This means, that c_{ij} is the share of the production of subsystem j, which will be send to subsystem i.

In the Section 3, we compare the implementation of the central planning, the QLE and the PHE in a certain scenario.

2.2. General networks and autonomous control with time-delays

In general production networks time-delays occur in form of transportation times. We take this into account by modeling production networks using retarded functional differential equations. The time needed for the transportation of material from the jth to the ith plant is denoted by $\tau_{ij} \in [0,\theta]$, where $\theta \in \mathbb{R}_+$ is the maximal involved delay. Furthermore, we consider disturbances in the transportation process from the jth to the ith plant, denoted by $d_{ij} \in [0,\Delta]$, where $\Delta \in \mathbb{R}_+$ is the maximal delay, which can be due to delay, for example, traffic jams, an accident or breakdowns of vehicles. Then, the dynamics of the ith subsystem can be described by

$$\dot{x}_i(t) = \sum_{j=1, j\neq i}^{n} c_{ij}(x(t-\tau_{ij}))\tilde{f}_j(x_j(t-\tau_{ij}-d_{ij}(t,\tau_{ij}))) + u_i(t) - \tilde{c}_{ii}(x(t))\tilde{f}_i(x_i(t)), \quad i = 1, \ldots, n. \tag{3}$$

The term $x_i^t := x_i(t-\tau_{ij}-d_{ij}(t,\tau_{ij})), x_i^t \in C([0, \theta+\Delta]; \mathbb{R})$ represents the state, where $C([0, \theta+\Delta]; \mathbb{R})$ denotes the space of continuous functions defined on $[0,\theta + \Delta]$ equipped with the norm $\|x_i^t\|_{[0,\theta+\Delta]} := \sup_{t\in[-\theta-\Delta,0]}|x_i(t)|$ and values in $\mathbb{R}$. We denote $x(t-\tau_{ij}) := (x_1(t-\tau_{ij}), \ldots, x_n(t-\tau_{ij}))^T$.

The external input and the processing rate do not depend on any time-delay and disturbance, but the internal inputs from other subsystems do so. This is represented by the terms $c_{ij}(x(t-\tau_{ij}))\tilde{f}_j(x_j(t-\tau_{ij}-d_{ij}(t,\tau_{ij})))$. This means, that the input of subsystem i at time t from subsystem j is the amount of material that was sent by the jth subsystem at the time $t-\tau_{ij}-d_{ij}(t,\tau_{ij})$. Note that the $c_{ij}(x(t-\tau_{ij}))$ do not depend on the disturbances $d_{ij}(t,\tau_{ij})$, because at the time the material will be send to a plant the disturbances in the future are unknown and cannot be taken into account for the calculation of the distribution rates. The influence of the disturbances on the stability of the system will be investigated in Section 5.

The distribution rates for time-delay systems are defined for the QLE by

$$c_{ij}^q(x(t-\tau_{ij})) := \frac{\frac{1}{x_i(t-\tau_{ij})+\varepsilon}}{\sum_k \frac{1}{x_k(t-\tau_{ij})+\varepsilon}},$$

where k is the index of the subsystems which get material from subsystem j. The distribution rates of the PHE are defined by

$$c_{ij}^{p}(x(t-\tau_{ij})) := (1-\upsilon_i)\frac{\tilde{f}_i(x_i(t-\tau_{ij}))}{\sum_k \tilde{f}_k(x_k(t-\tau_{ij}))+\varepsilon} + \sum_{k\neq i}\upsilon_k \frac{\tilde{f}_k(x_k(t-\tau_{ij}))}{\sum_q \tilde{f}_q(x_q(t-\tau_{ij}))+\varepsilon},$$

where k, q are indices of the subsystems which get material from subsystem j.

To illustrate this general modeling framework we consider a specific scenario of a production network in the following section.

3. Investigations on a certain scenario of a production network

In this section the presented modeling approach for production networks and autonomous control methods is applied to a certain scenario of a production network. We display advantages and disadvantages of the implementation of autonomous control methods in contrast to the central planning.

We consider a certain scenario of a production network as in Fig. 1, consisting of four plants. The first plant gets some raw material from an external supplier, denoted by u. At each plant the material will be processed with the rate $\tilde{c}_{ii}\tilde{f}_i, i = 1,\ldots,4$ and immediately send to the plants according to the network topology in Fig. 1. The first plant sends the production to the plants 2 and 3 corresponding to certain distribution coefficients c_{i1}. The plants 2 and 3 send the production to the plant 4, which will send one half of the produced material to some customers outside the network and one half back to the first plant. This can be interpreted as recycling, for example.

We choose $\tilde{c}_{ii} = 1$ and $\tilde{f}_i(x_i) = \alpha_i(1-\exp(-x_i))$, which is an increasing function and bounded by $\alpha_i \geqslant 0$. This choice takes into account the autonomous control of a plant. If the number of unprocessed parts within a plant is small, then the production rate is also small, i.e., dose to zero. If the number of parts increases, then the production rate increases up to the maximal possible production rate α_i.

Note that by different choices of $\tilde{f}_i(x_i)$ the systems behavior and the results of the (stability) analysis could be different. Our approach can be understood as a starting point of a mathematical analysis and to give statements about the systems

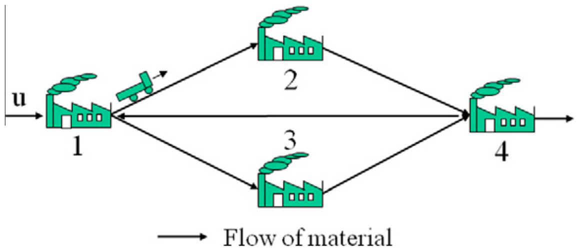

Fig. 1. Example of a scenario of a production network.

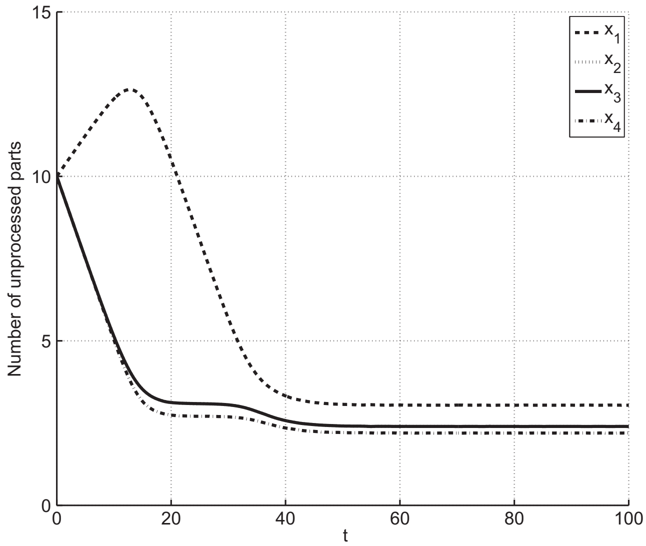

Fig. 2. Trajectories of the network scenario without transportation times.

behavior using autonomous control methods and time-delays. The stability analysis can be performed for general $\tilde{f}_i(x_i)$, but the choices of the gains are different for different $\tilde{f}_i(x_i)$ (see Chapter Section 4).

The differential equations that describe the systems behavior of the network without taking into account transportation times are of the form

$$\begin{aligned}
\dot{x}_1(t) &= u(t) + \tfrac{1}{2}\alpha_4(1-\exp(-x_4(t))) - \alpha_1(1-\exp(-x_1(t))),\\
\dot{x}_2(t) &= c_{21}\alpha_1(1-\exp(-x_1(t))) - \alpha_2(1-\exp(-x_2(t))),\\
\dot{x}_3(t) &= c_{31}\alpha_1(1-\exp(-x_1(t))) - \alpha_3(1-\exp(-x_3(t))),\\
\dot{x}_4(t) &= \alpha_2(1-\exp(-x_2(t))) + \alpha_3(1-\exp(-x_3(t))) - \alpha_4(1-\exp(-x_4(t))).
\end{aligned} \tag{4}$$

Three planning or control strategies are compared: by the choice of $c_{21} = c_{31} = \frac{1}{2}$ we implement a central planning, by $c_{i1} = c_{i1}^q$ the QLE and by $c_{i1} = c_{i1}^p$ the PHE, $i = 2,3$ (see the previous section). The following figure shows the trajectories of the queue lengths of unprocessed parts of the plants with the central planning implemented, where we choose as initial values $x_i(0) = 10$, $i = 1,\ldots,4$, a constant input $u(t) \equiv 10$ and $\alpha_1 = 21$, $\alpha_2 = \alpha_3 = 11$, $\alpha_4 = 22.5$.

The trajectories of the network with the implementation of the QLE or PHE are similar and we skip them. Note that by the choice of an fluctuating input $u(t) = 5(1 + \sin(t))$ one can simulate seasonal changes of the demand, for example, but we also skip this, because for the investigations and goals of this paper it is enough to consider constant inputs u (for fluctuating inputs, see [6,9]).

Now, we consider transportation times, represented as time-delays τ_{ij} in the network model as follows

$$\begin{aligned}
\dot{x}_1(t) &= u(t) + \tfrac{1}{2}\alpha_4(1-\exp(-x_4(t-\tau_{14}))) - \alpha_1(1-\exp(-x_1(t))),\\
\dot{x}_2(t) &= c_{21}\alpha_1(1-\exp(-x_1(t-\tau_{21}))) - \alpha_2(1-\exp(-x_2(t))),\\
\dot{x}_3(t) &= c_{31}\alpha_1(1-\exp(-x_1(t-\tau_{31}))) - \alpha_3(1-\exp(-x_3(t))),\\
\dot{x}_4(t) &= \alpha_2(1-\exp(-x_2(t-\tau_{42}))) + \alpha_3(1-\exp(-x_3(t-\tau_{43}))) - \alpha_4(1-\exp(-x_4(t))).
\end{aligned} \tag{5}$$

Again, $c_{21} = c_{31} = \frac{1}{2}$ represent the central planning and $c_{i1} = c_{i1}^q, c_{i1} = c_{i1}^p$ as defined in the previous section represent the QLE or PHE, respectively.

We choose the initial functions $x_i(\tau) \equiv 4$, $x_3(\tau) \equiv 1$, $i = 1,2,4$, $\tau \in [-\theta, 0]$, a constant input $u(t) \equiv 10$ and $\alpha_1 = 21$, $\alpha_2 = \alpha_3 = 11$, $\alpha_4 = 22.5$. For $\theta = 1$ the trajectories of the plants are displayed in the Fig. 3 using the central planning, in the Fig. 4 using the QLE and in Fig. 5 using the PHE with $v_i = 0.1$. Using autonomous control methods in networks under consideration of transportation times, we observe oscillating behaviors of the state of the plants 2 and 3 and caused by this also oscillating behaviors of the plants 1 and 4 in contrast to the central planning, where no such oscillating behaviors are observed.

Furthermore, it can be observed that the amplitude of the oscillating behavior using the QLE is higher in contrast to the usage of the PHE.

If we increase the transportation times, for example by choosing $\theta = 2$, then the trajectories are displayed in the Figs. 6–8.

We observe that the amplitude of the oscillating behavior is the higher the larger the transportation times are.

The reason of the appearance of the oscillating behaviors is that by the definition of c_{ij}^q and c_{ij}^p in the previous section we do not take into account the parts, which are currently in transit to a plant. It may happen that a lot of parts will be send to the second plant (and less material to the third one) according to the actual distribution rate, although there are a lot of parts in transit and will arrive during the next time period. When they arrive within the transportation time period $[t, t + \theta]$ the

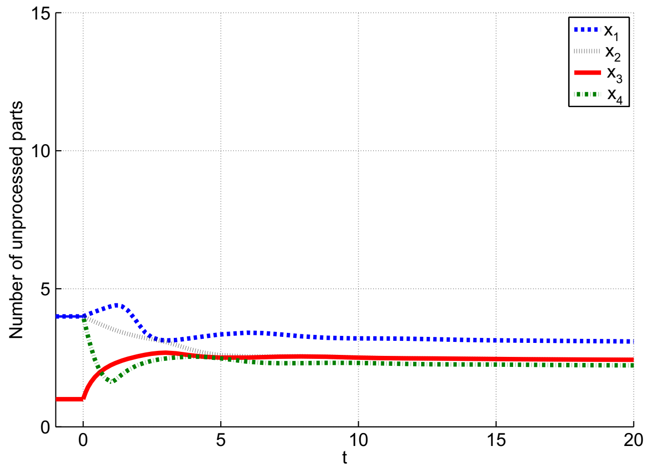

Fig. 3. Trajectories of the network scenario with central planning and transportation time 1.

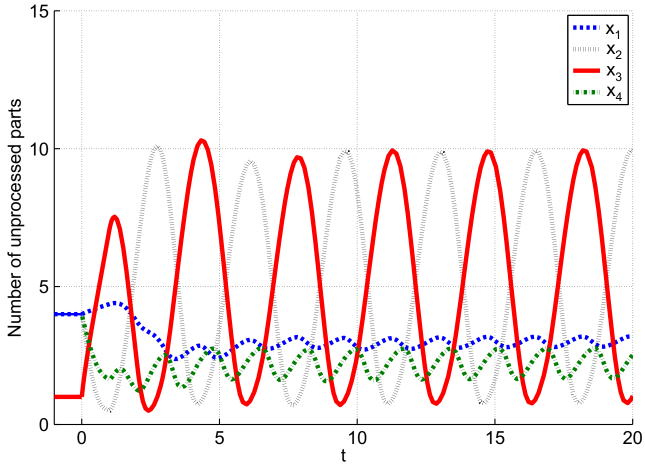

Fig. 4. Trajectories of the network scenario with QLE and transportation time 1.

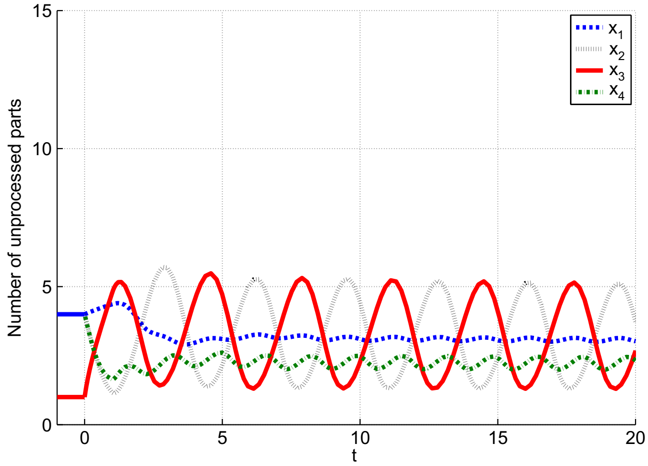

Fig. 5. Trajectories of the network scenario with PHE and transportation time 1.

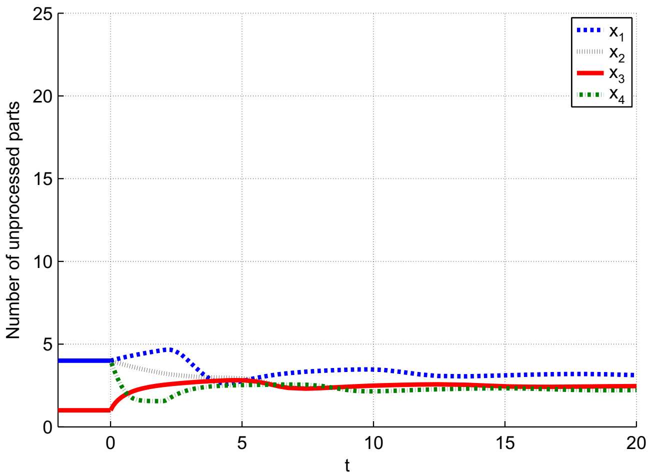

Fig. 6. Trajectories of the network scenario with central planning and transportation time 2.

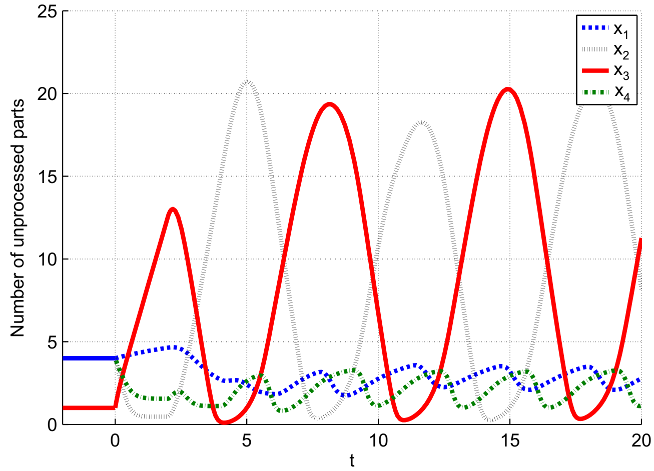

Fig. 7. Trajectories of the network scenario with QLE and transportation time 2.

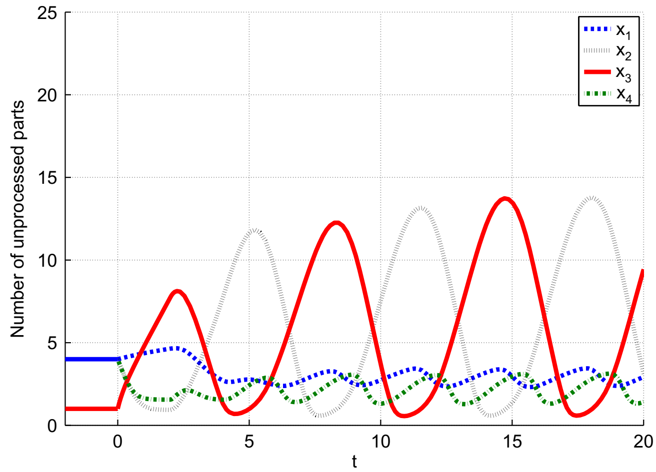

Fig. 8. Trajectories of the network scenario with PHE and transportation time 2.

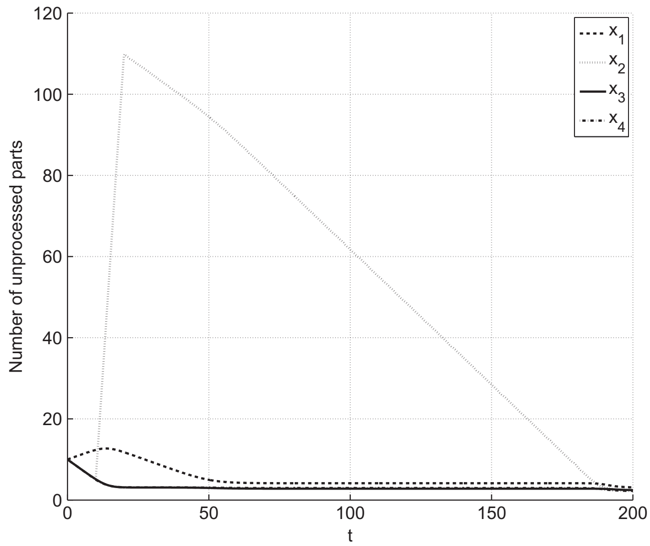

Fig. 9. Trajectories, considering a breakdown, with an implemented central planning.

queue length of the second plant increases, whereas the queue length of the third plant decreases, according to the production rates. At a certain time there is a turning point, where more parts will be send to the third plant than to the second one. From $t + \theta$ on this results in a decreasing queue length of the second plant and an increasing one of the third plant. Then this cycle will start again.

With this explanation it becomes clear why the amplitude is the higher the larger the time-delay is: parts will be transported a longer time period and therefore the number of parts which are in transit, but not taking into account by the distribution rates, is larger in contrast to a smaller time period.

To avoid the oscillating behaviors one can adapt the defined distribution rates of the QLE and PHE by taking into account the parts in transit by an additional term $\int_{t-\tau_{ij}-d_{ij}(t,\tau_{ij})}^{t} x_i(s)ds$, for example. In reality this requires full information access about material in traffic or disturbances, for example, which has to be ensured.

3.1. Breakdown of a plant

In this subsection we model a breakdown of a plant and compare the system behaviors using the central planning, QLE and PHE.

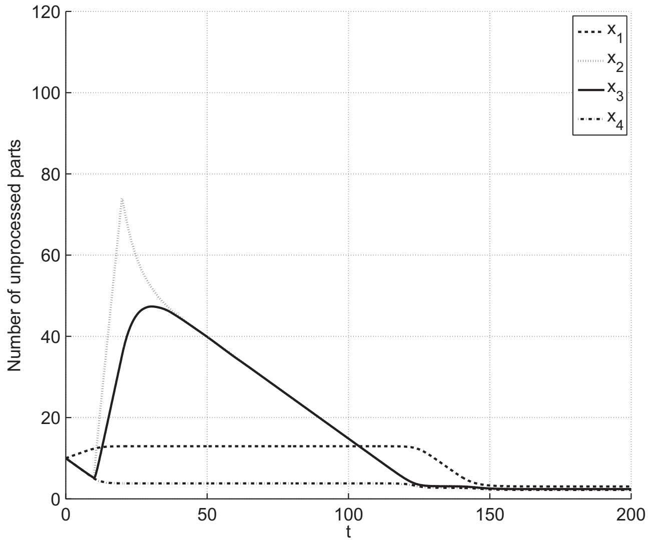

Fig. 10. Trajectories, considering a breakdown, with an implemented QLE.

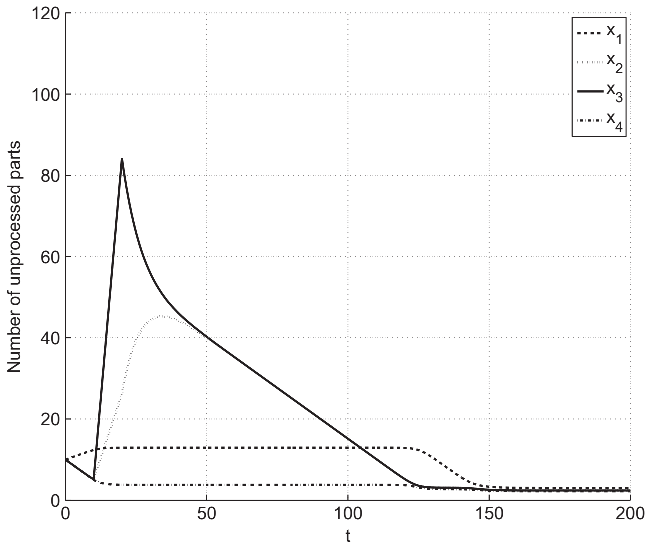

Fig. 11. Trajectories, considering a breakdown, with an implemented PHE with $\nu = 0.1$.

We consider the certain scenario of a production network as in Fig. 1 modeled by the Eq. (4). Here, we do not take into account transportation times, because we observed that they have no influence to the results or the conclusions of the comparison of the different control methods.

The scenario is adapted by the assumption of a breakdown of the second plant within the time interval from $t = 10$ to $t = 20$. This is modeled by setting the production rate equal to 0 in the interval [10,20]. In the Figs. 9–11 the trajectories of the plants are displayed implementing the central planning, the QLE and the PHE with $v_i = 0.1$, choosing the initial values $x_i(0) = 10$, $i = 1,\dots,4$, a constant input $u(t) \equiv 10$ and $\alpha_1 = 21$, $\alpha_2 = \alpha_3 = 11$, $\alpha_4 = 22.5$. It can be observed that the autonomous control methods can handle the breakdown of a plant in a better way, compared to a central planning: the maximum value of the queue length is smaller and less time is needed for a normalization of the queue lengths levels. The reason is that the QLE and the PHE can react to breakdowns by sending the parts to the third plant with a higher distribution rate, whereas the central planning cannot react so quickly and parts will be send to the second plant although it is not producing.

Parts are send to both plants according to the QLE and when the second plant is ready for production after the time $t = 20$ it can produce near by the maximal production rate. In total, the maximum of the queue lengths do not increase as much as

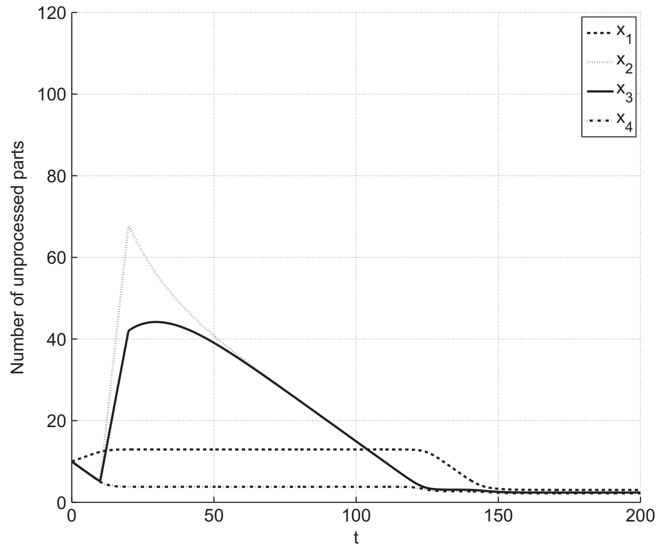

Fig. 12. Trajectories, considering a breakdown, with an implemented PHE with $v = 0.3$.

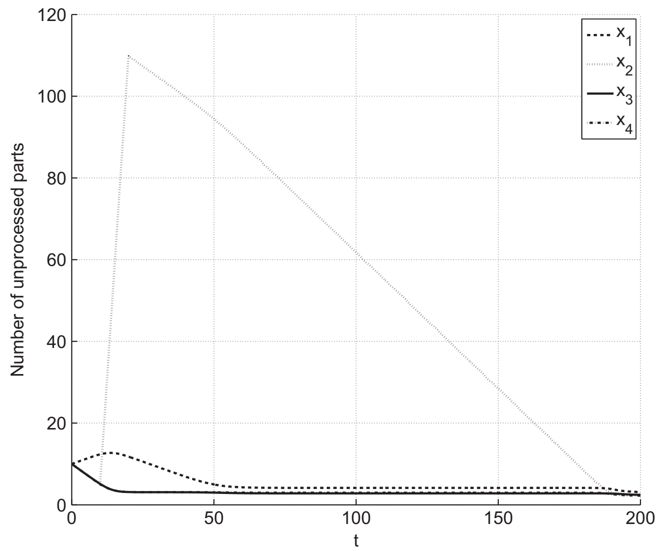

Fig. 13. Trajectories, considering a breakdown, with an implemented PHE with $v = 0.5$.

in the scenario with a central planning and return to the "normal" (without a breakdown) queue length level faster at the time $t = 150$ in opposite to $t = 180$ for the central planning.

Implementing the PHE with $v_i = 0.1$, the queue length of the third plant grows faster in the time interval from $t = 10$ to $t = 20$ than using the central planning or the QLE, where the state of the second plant grows faster. The maximum value of the number of parts using the PHE is higher than using the QLE, but smaller than using the central planning. The time needed to "normalize" the queue length levels is equal to the time needed to "normalize" the queue length levels using the QLE.

Choosing different v_i we observe different trajectories: for $v_i = 0.3$ the maximum queue length level is a bit less than the level using the QLE, but for $v_i = 0.5$ the maximum queue length level is as high as using the central planning and the time needed to "normalize" the queue length levels is also as high as using the central planning (see Figs. 12, 13). This shows, that one has to adapt the PHE (choosing different v_i) to achieve desired economic goals. We can conclude that the implementation of autonomous control methods makes the network more robust to breakdowns of plants in contrast to the usage of the central planning.

4. Stability analysis of general autonomously controlled production networks

In this section we investigate whether the implementation of autonomous control methods have an effect on the derived parameters, which guarantee stability of a network.

We note the stability property and a tool to verify whether a system of a network has this property. More details can be found in the mentioned papers [8–11] and the references therein.

In general, production networks consist of $n \in \mathbb{N}$ interconnected systems of the form

$$\dot{x}_i(t) = f_i(x_1(t), \dots, x_n(t), u_i(t)), \quad t \in \mathbb{R}_+, \quad i = 1, \dots, n, \tag{6}$$

where $x_i \in \mathbb{R}^{N_i}, u_i \in \mathbb{R}^{M_i}$ and $f_i : \mathbb{R}^{\sum_{j=1}^{n} N_j + M_i} \to \mathbb{R}^{N_i}$ are locally Lipschitz continuous functions. Here, $x_j, j \neq i$ can be interpreted as internal inputs of the ith subsystem and the solution is denoted by $x_i(t; x_i^0, x_j, j{\neq}i, u_i)$ or $x_i(t)$ for short, where $x_i^0 := x_i(0)$ is the initial condition.

If we define $N := \sum_{i=1}^{n} N_i, M := \sum_{i=1}^{n} M_i, x := (x_1^T, \dots, x_n^T)^T$, $u := (u_1^T, \dots, u_n^T)^T$ and $f = (f_1^T, \dots, f_n^T)^T$, then the interconnected system of the form (6) can be written as one single system of the form

$$\dot{x}(t) = f(x(t), u(t)), \quad t \in \mathbb{R}_+, \tag{7}$$

where $x \in \mathbb{R}^N$ denotes the state of the system, $u \in \mathbb{R}^M$ and $f : \mathbb{R}^N \times \mathbb{R}^M \to \mathbb{R}^N$ describes the system dynamics.

For the stability notions, we need the following classes of comparison functions:

$$\begin{aligned}
&\mathcal{P} := \{f : \mathbb{R}^n \to \mathbb{R}_+ | f(0) = 0,\ f(x) > 0,\ x \neq 0\},\\
&\mathcal{K} := \{\gamma : \mathbb{R}_+ \to \mathbb{R}_+ | \gamma \text{ is continuous}, \gamma(0) = 0 \text{ and strictly increasing}\},\\
&\mathcal{K}_\infty := \{\gamma \in \mathcal{K} | \gamma \text{ is unbounded}\},\\
&\mathcal{L} := \{\gamma : \mathbb{R}_+ \to \mathbb{R}_+ | \gamma \text{ is continuous and strictly decreasing with } \lim_{t\to\infty} \gamma(t) = 0\},\\
&\mathcal{KL} := \{\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+ | \beta \text{ is continuous},\\
&\beta(\cdot, t) \in \mathcal{K},\ \forall t \geqslant 0,\ \beta(r, \cdot) \in \mathcal{L},\ \forall r > 0\}.
\end{aligned}$$

We will call functions of class $\mathcal{P}$ *positive definite*.

We introduce the following stability notion:

Definition 4.1

1. System (7) is locally input-to-state stable (LISS) if there exist constants $\rho, \rho_u > 0, \gamma \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ such that for all initial values $|x_0| \leqslant \rho$ and all inputs $\|u\|_\infty \leqslant \rho_u$ the inequality

$$|x(t)| \leqslant \max\{\beta(|x_0|, t), \gamma(\|u\|_\infty)\}$$

is satisfied $\forall\ t \in \mathbb{R}_+$, where $|\cdot|$ denotes the Euclidean norm and $\|u\|_\infty := \text{esssup}_{t \in [0,\infty)} |u(t)|$ is the essential supremum norm. γ is called (nonlinear) gain.

2. The ith subsystem of (6) is called *LISS* if there exist constants $\rho_i, \rho_{ij}, \rho_i^u > 0,\ \gamma_{ij}, \gamma_i \in \mathcal{K}$ and $\beta_i \in \mathcal{KL}$ such that for all initial values $|x_i^0| \leqslant \rho_i$ and all inputs $\|x_j\|_\infty \leqslant \rho_{ij}, \|u_i\|_\infty \leqslant \rho_i^u$ the inequality

$$|x_i(t)| \leqslant \max\left\{\beta_i(|x_i^0|, t), \max_{j \neq i} \gamma_{ij}(\|x_j\|_\infty), \gamma_i(\|u_i\|_\infty)\right\}$$

is satisfied $\forall t \in \mathbb{R}_+$. γ_{ij} and γ_i are called (nonlinear) gains.

Note that, if $\rho, \rho_u = \infty$ then the system (7) is called (*global*) *ISS* and if $\rho_i, \rho_{ij}, \rho_i^u = \infty$ then the ith subsystem of (6) is called (global) ISS. In particular LISS and ISS guarantee that the norm of the trajectories of each subsystem is bounded.

An important tool to verify LISS and ISS, respectively, of a system of the form (6) are Lyapunov functions.

Definition 4.2. We assume that for each subsystem of the interconnected system (7) there exists a function $V_i : \mathbb{R}^{N_i} \to \mathbb{R}_+$, which is locally Lipschitz continuous. Then, for $i = 1,\dots,n$ the function V_i is called a *LISS Lyapunov function of the ith subsystem of (6)*, if V_i satisfies the following two conditions:

There exist functions $\psi_{1i}, \psi_{2i} \in \mathcal{K}_\infty$ such that

$$\psi_{1i}(|x_i|) \leqslant V_i(x_i) \leqslant \psi_{2i}(|x_i|),\ \forall\ x_i \in \mathbb{R}^{N_i} \tag{8}$$

and there exist $\gamma_{ij}, \gamma_i \in \mathcal{K}$, a positive definite function μ_i and constants $\rho_i, \rho_{ij},\ \rho_i^u > 0$ such that

$$V_i(x_i) \geqslant \max\left\{\max_{j\neq i} \gamma_{ij}(V_j(x_j)), \gamma_i(|u_i|)\right\} \Rightarrow \nabla V_i(x_i) \cdot f_i(x,u) \leqslant -\mu_i(V_i(x_i)) \tag{9}$$

for almost all $x_i \in \mathbb{R}^{N_i}, |x_i^0| \leqslant \rho_i, |x_j| \leqslant \rho_{ij}, u_i \in^{M_i}, |u_i| \leqslant \rho_i^u, \gamma_{ii} = 0$, where ∇ denotes the gradient of the function V_i. Functions γ_{ij} are called *LISS Lyapunov gains*.

Note that, if $\rho_i, \rho_{ij}, \rho_i^u = \infty$ then the LISS Lyapunov function of the *i*th subsystem becomes an ISS Lyapunov function of the *i*th subsystem (see [12]). In general, the LISS Lyapunov gains are different from the gains in Definition 4.1.

Condition (8) implies that V_i is positive definite and radially unbounded. V_i can be interpreted as the energy of a system and the second condition (9) of a Lyapunov function means that if $V_i(x_i) \geqslant \max\{\max_{j\neq i}\gamma_{ij}(V_j(x_j)), \gamma_i(|u_i|)\}$ holds, then the energy decreases. If $V_i(x_i) < \max\{\max_{j\neq i}\gamma_{ij}(V_j(x_j)), \gamma_i(|u_i|)\}$ then the energy of the system is bounded by the expression on the left side of the previous inequality. Overall, the trajectory of a system is bounded.

Furthermore, we define the *gain-matrix* $\Gamma := (\gamma_{ij})_{n\times n}$, $i,j = 1,\dots,n$, $\gamma_{ii} = 0$, which defines a map $\Gamma : \mathbb{R}_+^n \to \mathbb{R}_+^n$ by

$$\Gamma(s) := \left(\max_j \gamma_{1j}(s_j), \dots, \max_j \gamma_{nj}(s_j)\right)^T, \quad s \in \mathbb{R}_+^n. \tag{10}$$

Note that the matrix Γ describes in particular the interconnection structure of the network, moreover, it contains the information about the mutual influence between the subsystems, which can be used to verify the (L) ISS property of networks.

To analyze the whole network in view of stability we need a small-gain condition (see [13]):

Definition 4.3. Γsatisfies the *local small gain condition (LSGC) on [0,w*]*, provided that

$$\Gamma(w^*) < w^* \quad \text{and} \quad \Gamma(s) \not\geqslant s,\ \forall s \in [0, w^*], \quad s \neq 0. \tag{11}$$

The notation $\not\geqslant$ means that there is at least one component $i \in \{1,\dots,n\}$ such that $\Gamma(s)_i < s_i$.

To check whether the interconnected system of the form (7) has the LISS property, one has to find a LISS Lyapunov function for each subsystem. If it exists then this subsystem has the LISS property. Furthermore, if the LISS Lyapunov gains satisfy the local small-gain condition (4.3), then the whole system of the form (7) is LISS, which we recall in the following theorem (see [13]):

Theorem 4.1. *Consider the interconnected system (6), where each subsystem has an LISS Lyapunov function V_i. If the corresponding gain-matrix Γ satisfies the local small-gain condition (11), then there exist constants ρ, $\rho_u > 0$, such that the whole system of the form (7) is LISS.*

In [14,12] a similar ISS small-gain theorem for general networks was proved, where the small-gain condition is of the form

$$\Gamma(s) \not\geqslant s,\ \forall\ s \in \mathbb{R}_+^n \setminus \{0\}.$$

For time-delay systems one can define the notion of (L) ISS, a (L) ISS Lyapunov function and the result in Theorem 4.1 in a similar way (see [15]).

4.1. Results of the stability analysis of the certain scenario of a production network

We consider the same scenario as in the previous section (see Fig. 1) without transportation times. The details of the stability analysis are skipped here, but the calculations are similar to the analysis presented in [8,9]. We only present the results of the analysis. Using the model of the certain scenario (4) or with transportation times (5) implementing the central planning $(c_{21} = c_{31} = \frac{1}{2})$ we obtained, that for

$$\alpha_1 > \|u\|_\infty + \frac{1}{2}\alpha_4, \quad \alpha_2 > \frac{1}{2}\alpha_1, \quad \alpha_3 > \frac{1}{2}\alpha_1, \quad \alpha_4 > \alpha_2 + \alpha_3 \tag{12}$$

the subsystems and the network has the LISS property (states are bounded), if no breakdowns of plants occur. For example, for the choices $u \equiv 10$, $\alpha_1 = 21$, $\alpha_2 = \alpha_3 = 11$, $\alpha_4 = 22.5$ and $x_i^0 = 10$ implementing in the network model without time-delays we obtain the stable behavior displayed in Fig. 2. If we choose $\alpha_1 = 19$, $\alpha_2 = \alpha_3 = 9.6$, $\alpha_4 = 19.5$ and $x_i^0 = 10$ such that the

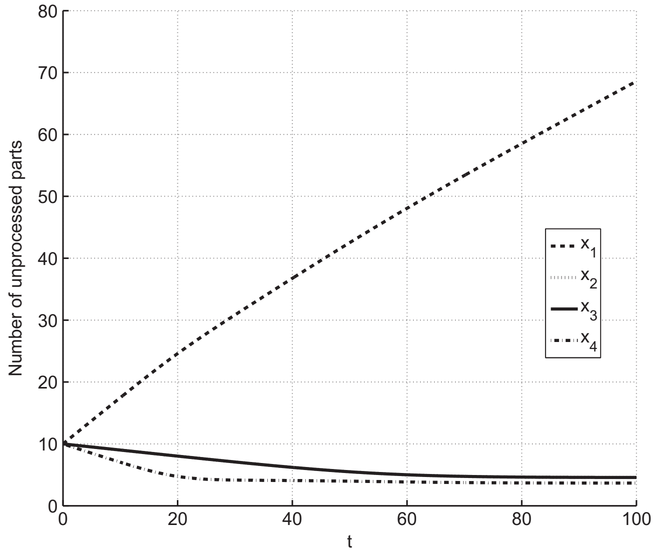

Fig. 14. Trajectories of the network scenario - unstable behavior.

conditions (12) are not satisfied, then we obtain the unstable behavior of the network as shown in Fig. 14. The stability analysis of implementing the QLE or PHE results in the conditions

$$\alpha_1 > \|u\|_\infty + \frac{1}{2}\alpha_4, \quad \alpha_2 > \alpha_1, \quad \alpha_3 > \alpha_1, \quad \alpha_4 > \alpha_2 + \alpha_3. \tag{13}$$

For a given input u the set of parameter of α_i, which satisfy the conditions in (13) is less than the set of parameters, which satisfy the conditions (12) calculated using the central planning. The reason is, that within the presented stability analysis the worst case is taken into account. For the QLE or PHE the "worst case" is that the distribution rates are equal to one, which means that a plant gets all of the production from the supplier plant. In the central planning scenario there are fixed distribution rates.

If we consider a breakdown of a plant, then the system behavior may become unstable in the time interval of the breakdown, if only the conditions to the parameters α_i derived from the implementation of a central planning are fulfilled. In opposite, if the conditions to α_i derived from implementing an autonomous control method are fulfilled, then stability can be guaranteed for the time of the breakdown also.

Finally, as a last result from the stability analysis, we observe that for time-delays $\tau_{ij} < \tilde{\tau}_{ij}$ the estimation of the state may be higher. This is the case, because in the definition of LISS (Definition 4.1) the estimation is the higher the (internal and external) inputs are, i.e., for $\tau_{ij} < \tilde{\tau}_{ij}$ the estimation may be higher. For production networks this means that the maximum queue length level is the higher the transportation time is, which was observed in the Figs. 4–8. This is also the case for disturbances in the transportation time: the estimation of the state is the higher the disturbance is.

Note that by the help of the functions γ_{ij} we can calculate explicitly the "long-term" bound, i.e., the "long-term" maximum queue length level of the subsystems of a network for given initial values. Together with the information about the parameters α_i from the stability analysis this can be used, for example, to design stable networks with suitable production rates (suitable: choices of α_i to assure stability and the prevention of over capacities in the production rates) and with suitable inventory levels (suitable: in view of the size of the inventory to avoid high costs caused by the over dimension and vacancy of the inventory).

5. Discussion of implementing different control methods in production networks

In this section we collect and discuss the advantages/disadvantages of implementing autonomous control methods in contrast to a central planning, which were shown in the previous sections.

The observations about the advantages/disadvantages were made by the investigation of a certain scenario of a production network. So, this discussion or the conclusions given here may be not true for general networks, since at the moment we cannot prove the advantages/disadvantages for general networks. But the case of decision making to which plant parts will be send, as in the certain scenario, appears in general large-scale networks very often such that one can assume that the advantages/disadvantages occur at every stage in a general large-scale network with such a case of decision making.

A disadvantage of implementing autonomous control methods in contrast to a central planning is that taking into account transportation times one observes an oscillating behavior of the systems state (the queue lengths), although there is no oscillating input. The amplitude of the trajectories is the higher the transportation time is. This effect can be avoided by taking into account the parts in transit in the modeling of the autonomous control methods.

An advantage is that the usage of autonomous control makes the system more robust to breakdowns of plants or machines. The autonomous control method can react quickly to such breakdowns and can send parts to other plants or machines according to the distribution rates of the autonomous control method. This results in a lower maximum value of the queue length and less time needed for a "normalization" of the queue length levels after the ended breakdown.

In view of stability of a production network the set of parameters, which guarantee stability, implementing an autonomous control method is less then the set by implementing a central planning. The reason is the "worst-case" approach of the stability analysis. This has the advantage, that the set of parameters obtained by the implementation of autonomous control methods guarantees stability of the network in case of breakdowns of plants or machines for all times (even during the time interval of the breakdown, which is not the case for the central planning).

6. Summary

6.1. Conclusions

We modeled general production networks by differential equations, where transportation times and disturbances of the transportation times were considered. Different autonomous control strategies (the QLE and the PHE) were modeled and implemented in the network model. Based on this, we simulated and investigated a certain scenario of a production network. We analyzed the influence of transportation times and disturbances in the transportation times to the queue length levels of the plants and simulated a breakdown of a plant. It was shown that the usage of autonomous control methods is more robust in contrast to the usage of a central planning. Finally, a stability analysis was performed and the advantages/disadvantages of the implementation of autonomous control in comparison to a central planning were discussed.

6.2. Future work

One can investigate in more detail the role of the evaporation constant v_i and one can analyze in which situation a certain choice of v_i results in optimal trajectories in view of robustness or performance. The availability of full information about system states to avoid an oscillating behavior of the system trajectories can be investigated and the autonomous control methods can be adapted.

In networks, for example a shop-floor with a large number of machines and a high degree of automation, it may happen that by implementing autonomous control methods in the network, machine breakdowns will not be detected or not quickly detected. Parts will be further processed using other machines according to the autonomous control method, which means that the production runs and everything seems alright, but the performance of the system decreases by an increased number of machine breakdowns. Therefore, it is worth investigating an implementation of fault detection for autonomous controlled systems, modeled by nonlinear differential equations. Using this detection a machine breakdown will be quickly detected and the machine can be repared, for example.

In this paper we consider only one type of material. The next step is to model production networks, which process different types of material and analyze the implemenation of autonomous control methods in view of robustness, performance and stability according to [6].

Finally, the statements and the conclusions made in Section 5 can be proved for general networks. One has to search for (mathematical) tools, which are useful for their proofs.

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