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Stability analysis of autonomously controlled production networks

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In this paper we present a stability analysis of autonomously controlled production networks from mathematical and engineering points of view. Roughly speaking stability of a system means that the defined state of the system remains bounded over time. The dynamics of a production network are modelled by differential equations (macroscopic approach) and discrete event simulation (microscopic approach), respectively. Both approaches are used to perform a stability analysis. As a result of the stability analysis of the macroscopic approach we calculate parameters, which guarantee stability of the network for arbitrary inputs. These results are refined for a certain (varying) input using the microscopic approach, where we derive the smallest maximal production rates of the plants for which stability of the overall system can be guaranteed. Furthermore, the microscopic approach includes two different autonomous control methods: the queue length estimator (QLE) and the pheromone based (PHE) method. These methods allow additional autonomous decision making on the shop floor level. The approach presented in this paper is to calculate stability conditions by mathematical systems theory to guarantee stability for production networks, to identify a stability region and to refine this region by simulations.

Keywords: production networks; stability analysis; simulation

1. Introduction

Modern logistic systems are exposed to various dynamical changing parameters in its internal and external environment. Especially logistic networks, e.g., production networks or whole supply chains, may be affected by dynamical changes (Stadtler 2005, Sydow 2006). These dynamics may be induced for example by uncertainties of demand, the desire of customers for individualised products or internal disturbances such as machine failures. Due to the high degree of structural and dynamic complexity interconnected networks of logistic systems, i.e., production networks, may exhibit unexpected and unfavourable system behaviour, in terms of increasing throughput times, increasing tardiness of orders or underutilisation of resources (Min and Zhou 2002, Rabelo *et al.* 2008).

The implementation of decentralised approaches such as autonomous control opens new potentials to cope with increasing external and internal dynamics. The concept of autonomous control aims at increasing the robustness and the performance of logistic systems (Windt 2006, Windt and Hülsmann 2007). Autonomous control enables single intelligent logistic objects (parts, machines, orders) to make and execute decisions, based

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on local information, on their own. However, this kind of autonomous decision making causes a decentralised system behaviour, which may affect the logistic performance negatively or even lead to instability of the system (Windt 2006, Philipp *et al.* 2007).

Thus, this paper focuses on the stability analysis of autonomously controlled production networks, in order to identify parameters, which guarantee stability of the network. Roughly speaking, stability means that the state of a plant remains bounded over time, whereas instability of a network leads to infinite states. The state variables describe certain processes of a plant. The present state and the chosen input/control function together determine the future states of the plant. More precisely, given the initial state $x(t_0) = x_0$ of the plant at some initial time t_0 and an input/control $u(\cdot)$, the evolution of the plant's state $x(t)$ is uniquely determined for all t in a suitable time interval and the state only depends on the input/control values.

In this paper, the state of a plant represents the work in progress (WIP). Instability, by means of an unbounded growth of a state, may cause high inventory costs or loss of customers. Hence, it is necessary for logistic systems to derive parameters, which guarantee stability. By applying a stability analysis to a logistic network we can draw conclusions of its behaviour and derive parameters to guarantee stability, which avoid such negative outcomes.

To demonstrate the application of a stability analysis we consider a certain autonomously controlled production network scenario with six interconnected plants. All plants comprise a shop floor with parallel processing lines on different production stages. On this shop-floor level two different local autonomous control methods are implemented.

Existing works on the stability of autonomously controlled production networks showed that parameters for varying production rates, which guarantee stability, can be calculated by the help of a model, which is based on a system of differential equations (Scholz-Reiter *et al.* 2005a, Dashkovskiy *et al.* 2009). This paper also uses this modelling approach, in order to derive production rates, which guarantee stability of a production network. Due to the high complexity of the chosen scenario this mathematical approach is applied to the macroscopic network level. Subsequently, the stability parameters, calculated by the mathematical stability analysis are implemented into a more detailed microscopic model, where all plants are represented by a complete shop floor. This microscopic view models the scenario with the help of a discrete event simulation (DES) tool. A comparison of the mathematical stability analysis and the simulation results from the DES model gives information about the stability regions from both perspectives (macroscopic and microscopic). The advantage is that we first apply the mathematical theory to find in a very fast way those parameters, where stability is guaranteed. Subsequently, a refinement is performed by simulations, in order to enlarge the set of parameters, which guarantee stability. A simulation of the model, based on differential equations is conducted additionally, in order to verify the comparability of the macroscopic mathematical model and the microscopic DES model.

To achieve the aim of this paper, i.e., the identification of stability regions for autonomously controlled production networks, this work is structured as follows. Section 2 presents the concept of autonomous control in the context of production logistics. An overview about production networks and stability of logistic systems is given in Section 3. Section 4 presents the mathematical approach of a stability analysis of general networks and a procedure to perform a stability analysis. According to this procedure we model a concrete scenario in Section 5. A general description of the scenario is presented in Section 5.1. This is followed by a detailed description of all relevant modelling parameters, concerning the mathematical model (Section 5.2) and the DES model (Section 5.3). In Section 6 the mathematical stability analysis of the particular scenario depicted in Section 5.2 is presented. There, the procedure in Section 4 is applied to the scenario to derive stability conditions. Subsequently, Section 7 shows the simulation results and the refinement of the macroscopic approach by the microscopic approach. Finally, Section 8 gives a summary and an outlook concerning further research directions.

2. Autonomous control

The approach of autonomous control, coming from the theory of self-organisation, aims at enabling intelligent logistic objects to gather relevant local information, to render and to execute decisions on their own (Windt *et al.* 2008). In this context intelligent logistic objects may be physical or material objects, e.g., parts or machines, as well as immaterial objects (e.g., production orders, information). The use of modern information and communication technologies enables these objects to interact with others. Based on these interactions, logistic objects collect information about the current local system states and use this information for the decentralised decision making. These autonomous and decentralised decisions affect the dynamic behaviour of a logistic system (Windt *et al.* 2005).

The general idea of autonomous controlled logistic processes is to influence the dynamical systems behaviour positively. Due to the complex interactions between autonomously acting objects, the evaluation of such systems should not be limited to a pure analysis of classical performance measures. Moreover, an investigation of the dynamical behaviour of the total system is required (Scholz-Reiter and Freitag 2007). In the context of production logistics, investigations on the performance and the dynamical behaviour of autonomously controlled systems showed an increased performance and robustness compared to conventional planned systems (Scholz-Reiter *et al.* 2005b, 2007). The analysis of a real data based manufacturing case confirmed furthermore, that an increase of the degree of autonomy improves the handling of the dynamic complexity, compared to a centralised production planning and control approach (Scholz-Reiter *et al.* 2009a).

However, these studies showed that the performance of different autonomous control methods depends on the particular scenario and the corresponding internal and external systems parameters (Hülsmann *et al.* 2008). Unfavourable parameter constellations may cause sudden changes of the systems behaviour and lead to a worse logistic performance. This can result in increasing throughput times or growing inventory. Therefore, the application of mathematical methods, e.g., the stability analysis, can be used to obtain reliable information about the autonomous systems behaviour.

Larger logistic systems, i.e., supply chains or production networks, are characterised by a high degree of structural and dynamical complexity. Conventional incremental planning and control methods have shortcomings to cope with such challenges (Ivanov 2010). Hence, the application of autonomous control is suitable for these kinds of logistic networks. As far as production networks are concerned, first autonomous control approaches were formulated (Scholz-Reiter *et al.* 2009b). Autonomous control in production networks can improve the networks performance and robustness, as well. On the other hand, beyond limiting parameters constellations, unpredictable and unstable system behaviour can be observed (Scholz-Reiter *et al.* 2009b). Thus, this paper focuses on the application of mathematical stability analysis methods on autonomously controlled production networks and the identification and refinement of stability regions.

3. Stability of production networks

The term production network is used to describe company or cross-company owned networks with geographically dispersed plants. The primary objective of production networks is to archive economies of scale through joint planning of production processes, a mutual use of common resources and an integrated planning of value added processes (Wiendahl and Lutz 2002). These types of networks may react quickly on perturbations due to redundancies of common resources. The redundancies allow the reorganisation of the material flow through the entire network in order to react to external or internal disturbances. According to this definition, new tasks and challenges appear in production networks: enterprises are forced to generate concepts for tasks like the choice of new partners, design of the network, product development and production planning and control (Sydow 2006). The high flexibility of these networks causes interdependencies between production processes in different plants, e.g., allocation problems for products, which can be processed in different plants or planning of transport and transport capacity (Sauer 2006, Alvarez 2007). Therefore, production planning and control (PPC) of production networks has to cover these new tasks additionally to the conventional functionalities. It should provide methods for an integrated planning and the synchronisation within the network, including planning of sales and inventory (Wiendahl and Lutz 2002). Under highly dynamic and complex conditions current PPC methods cannot cope with disturbances or unforeseen events in an appropriate manner (Kim and Duffie 2004). This may cause uncertainties of lead times, nervousness of schedules or may also lead to instability or even chaos.

Thus, the identification of stability regions is crucial in general for planning and operating logistic networks. In this context mathematical models are often used to determine stability regions. Queuing models can be used to analyse systems with multiple parallel servers. Whitt (1986) presents a two parallel server model without capacity restrictions at the queues and with service distributions, which are independent from the arrival process. It is shown that for certain distributions of service times the decision policy 'join the shortest queue' leads to suboptimal systems behaviour. Sharifnia (1997) presents a single station multi server model with multiple job classes. He compares the 'join the shortest queue' and the 'fist come first served policy' with regard to the systems stability. Sharifnia (1997) proposes guided policies, in order to stabilise the systems behaviour.

For manufacturing systems parameters, which guarantee stability, can also be found by using fluid models (Dai 1995). The stability can be analysed even for more complex systems using fluid models, e.g., manufacturing systems with re-entrant lines (Dai and Weiss 1996) or manufacturing systems with different job types and re-entrant lines (Dai and Vande Vate 2000). An approach with flows of multiple fluids was used to analyse the stability region of an autonomously controlled shop floor scenario by Scholz-Reiter *et al.* (2005a).

Scholz-Reiter *et al.* (2009c) present a fluid model of a production network and obtained a stability region from the stability analysis for a scenario with two locations and three types of products. First approaches have already been done to determine the stability region of autonomously controlled production networks by Dashkovskiy *et al.* (2009). Here a model, based on differential equations, is used to identify parameter constellations of variable production rates, which guarantee stability of an autonomously controlled production network, where Lyapunov functions for the stability analysis are used.

This paper focuses on the investigation of the refinement of the mathematically identified stability region of a production network, using the approach in Dashkovskiy *et al.* (2009), by simulations with the help of a DES model.

4. Mathematical background of the stability analysis

In this section we describe a general mathematical stability analysis method, related to ordinary differential equations, which are used in this paper to model production networks.

From a mathematical point of view production networks are non-linear interconnected dynamical systems. In mathematical systems theory the notion of input-to-state stability (ISS), introduced by Sontag (1989), has proved to be an efficient tool for the qualitative description of stability of non-linear dynamical control systems.

Definition: A dynamical system of the form $\dot{x}(t) = f(x(t), u(t))$, where t is the (continuous) time, $x(t) \in \mathbb{R}^N$ is the state, $u(t) \in \mathbb{R}^m$ is the input and $f: \mathbb{R}^{N+m} \to \mathbb{R}^N$ is a function, which is locally Lipschitz continuous in x uniformly in u, is called *input-to-state stable* (*ISS*), if there exist a function $\beta \in KL$ and a function $\gamma \in K$, such that for all initial values x_0 and all $t \geq 0$ it holds:

$$\|x(t)\| \leq \max\{\beta(\|x_0\|, t), \gamma(\|u\|_\infty)\},$$

where $\| \bullet \|$ denotes the Euclidean norm, $\|u\|_\infty$ is the supremum norm, roughly speaking the maximal value of $u(t)$. A function of class K is continuous, zero at zero and monotone increasing and a function of class KL is continuous, of class K in the first argument and monotone decreasing and tends to zero in the second argument. The function $\gamma \in K$ is called *gain*.

In simple words ISS means that the norm of the trajectory of a system is bounded over the time t. If t grows to infinity, the term $\beta(\|x_0\|, t)$ in the definition of ISS tends to zero. Since $\gamma(\|u\|_\infty)$ is a positive real value (because $\|u\|_\infty$ is a positive real value and $\gamma \in K$) the bound for trajectories given in the definition of ISS tends to $\gamma(\|u\|_\infty)$, if $t \to \infty$. This means that the ISS estimation for large time t depends essentially on γ and $\|u\|_\infty$: the larger $\|u\|_\infty$ is, the larger is the bound for the time dependent state $x(t)$.

The interpretation of the gain γ and therefore the value $\gamma(\|u\|_\infty)$ in view of production networks is the following: it quantifies the amount of the WIP of a plant depending on $\|u\|_\infty$ for all times. $\gamma(\|u\|_\infty)$ is the bound for the trajectories for large times, i.e., a bound for the WIP.

A helpful tool to verify whether a system has the ISS property is a Lyapunov function.

Definition: A smooth function $V: \mathbb{R}^N \to \mathbb{R}_+$ is called an *ISS Lyapunov function*, if it satisfies the following two conditions:

(1) There exist functions $\psi_1, \psi_2 \in K_\infty$ such that:

$$\psi_1(\|x\|) \leq V(x) \leq \psi_2(\|x\|), \quad \forall x \in \mathbb{R}^N.$$

(2) There exist $\chi \in K_\infty$, and $\alpha \in P$ such that:

$$V(x) \geq \chi(\|u\|) \Rightarrow \dot{V}(x) = \nabla V(x) \cdot f(x,u) \leq -\alpha(V(x)),$$

for all $t \in \mathbb{R}_+$, $x \in \mathbb{R}^N$, $u \in \mathbb{R}^m$ where ∇ denotes the gradient of V, a function of class K_∞ is a class K function and additionally unbounded and a function of class P is continuous, zero at zero and positive else. The function χ is called *ISS Lyapunov gain*.

Note that the Lyapunov gain and the gain in the definition of ISS are different in general. The condition (1) states, that V is positive, zero at zero and radially unbounded. By condition (2) the function V decreases (the full derivate of V along the trajectory is negative), whenever V is greater or equal than $\chi(\|u\|)$ and the term $\chi(\|u\|)$ can be interpreted as a bound for $V(x)$ for large times t.

It was shown by Sontag and Wang (1995) that the existence of a Lyapunov function is necessary and sufficient to guarantee ISS. The construction of an ISS Lyapunov function of a whole system consisting of interconnected non-linear control systems in terms of the ISS Lyapunov functions of the subsystems was shown by Jiang *et al.* (1996) (two subsystems) and by Dashkovskiy *et al.* (2007a, 2010) (general networks) under a small-gain condition. In the following we define the ISS property and the ISS Lyapunov functions for the subsystems of $n \in \mathbb{N}$ interconnected systems of the form:

$$\dot{x}_i(t) = f_i(x_1(t), \ldots, x_n(t), u_i(t)), \quad i = 1, \ldots, n. \tag{1}$$

Definition:

(1) The ith subsystem of (1) is called ISS, if there exist $\gamma_{ij}, \gamma_i \in K_\infty$, $j = 1, \ldots, n$, $j \neq i$ and $\beta_i \in KL$, such that for all initial values x_i^0 and all inputs u_i the inequality:

$$\left\| x_i\left(t; x_i^0, x_j : j \neq i, u_i\right) \right\| \leq \max\left\{ \beta_i\left(\left\|x_i^0\right\|, t\right), \max_{j \neq i} \gamma_{ij}\left(\left\|x_j\right\|_\infty\right), \gamma_i(\|u_i\|_\infty) \right\}, \tag{2}$$

is satisfied $\forall t \in \mathbb{R}_+$. γ_{ij} and γ_i are called (non-linear) gains.

(2) A smooth function $V_i : \mathbb{R}^{N_i} \to \mathbb{R}_+$ is called an *ISS Lyapunov function of the ith subsystem of system (1)*, if it satisfies the following two conditions:

(a) There exist functions $\psi_{1i}, \psi_{2i} \in K_\infty$ such that:

$$\psi_{1i}(\|x_i\|) \leq V_i(x_i) \leq \psi_{2i}(\|x_i\|), \quad \forall x_i \in \mathbb{R}^{N_i}.$$

(b) There exist χ_{ij}, $\chi_i \in K_\infty$, and $\alpha_i \in P$ such that:

$$\begin{aligned} &V_i(x_i) \geq \max\left\{\chi_{ij}\left(\left\|x_j\right\|\right), \chi_i(\|u_i\|)\right\} \\ &\quad \Rightarrow \dot{V}_i(x_i) = \nabla V_i(x_i) \cdot f_i(x_1, \ldots, x_n, u_i) \leq -\alpha_i(V_i(x_i)), \end{aligned}$$

for all $t \in \mathbb{R}_+$, $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{M_i}$, $i,j = 1, \ldots, n$. The functions χ_{ij}, χ_i are called *ISS Lyapunov gains*. Furthermore we define the *gain matrix* $\Gamma := (\chi_{ij})$, $i,j = 1, \ldots, n$, $\chi_{ii} = 0$ by:

$$\Gamma(s) := \left(\max\left\{\chi_{1j}(s_j)\right\}, \ldots, \max\left\{\chi_{nj}(s_j)\right\}\right), \quad s \in \mathbb{R}_+^n, j = 1, \ldots, n. \tag{3}$$

The *small-gain condition* used in Dashkovskiy *et al.* (2007a, 2010) is of the form:

$$\Gamma(s) \not\geq s, \quad \forall s \in \mathbb{R}_+^N \backslash \{0\}.$$

Notation $\Gamma(s) \not\geq s$ means that there is at least one component $i \in \{1, \ldots, n\}$ such that $\Gamma(s)_i < s_i$. Further information about the small-gain condition can be found in Dashkovskiy *et al.* (2007a, 2010). Now we can formulate the following theorem, which was proved by Dashkovskiy *et al.* (2010).

Theorem 1: *Suppose all the subsystems of* (1) *are ISS, i.e.,* (2) *holds for each* $i \in \{1, \ldots, n\}$ *or equivalently all subsystems have an ISS Lyapunov function. Suppose that the gain-matrix as defined in* (3) *satisfies the small-gain condition. Then the whole system of the form* $\dot{x}(t) = f(x(t), u(t))$ *is ISS, where* $N := \sum_{i=1}^{n} N_i$, $m := \sum_{i=1}^{n} M_i$, $x = (x_1^T, \ldots, x_n^T)^T$, $u := (u_1^T, \ldots, u_n^T)^T$ *and* $f := (f_1^T, \ldots, f_n^T)^T$.

This theorem states a stability condition for arbitrary interconnections of systems of the form (1). A local version of ISS considering interconnected systems can be found in Dashkovskiy *et al.* (2007b, 2010), called local input-to-state stability (LISS), where 'local' means that the estimation in the definition of ISS holds only for initial values x_i^0 and inputs u_i of the subsystems which satisfy $\|x_i^0\| < \rho_i$, $\|u_i\|_\infty < \rho_i^u$ for positive constants ρ_i and ρ_i^u, $i = 1, \ldots, n$. The previous mentioned results for ISS were extended to LISS (see Dashkovskiy *et al.* 2007b, 2010).

Summarising, to perform a stability analysis of a system, using the mathematical stability theory described in this section, we have to process the following steps. At first the system has to be modelled. Then the Lyapunov functions and the Lyapunov-gains for the subsystems have to be chosen. The Lyapunov- and the small-gain conditions have to be satisfied, checking this is the next step. If the conditions are satisfied under specific stability conditions, then the ISS or LISS property of the whole system is guaranteed by Theorem 1. Otherwise one has to choose other Lyapunov functions and gains and start the procedure again. If one cannot find sufficient functions, then no statement about stability is possible. The flowchart in Figure 1 depicts this procedure.

In the following sections we show the application of the mathematical stability theory for a certain scenario and process the procedure step by step according to Figure 1.

5. Modelling of a certain production network scenario

In this section a certain production network scenario is presented, which will be analysed in view of stability in the next section. Therefore, Section 5.1 presents the general structure of the network. The mathematical modelling approach for the macroscopic view is presented in Section 5.2. Subsequently, Section 5.3 introduces the microscopic modelling approach, using a DES model.

5.1 *Production network scenario*

The production network consists of six geographically dispersed plants. The *state* of each plant is denoted by $x_i(t)$, which are real values for $i = 1, \ldots, 6$, where t is also a real value

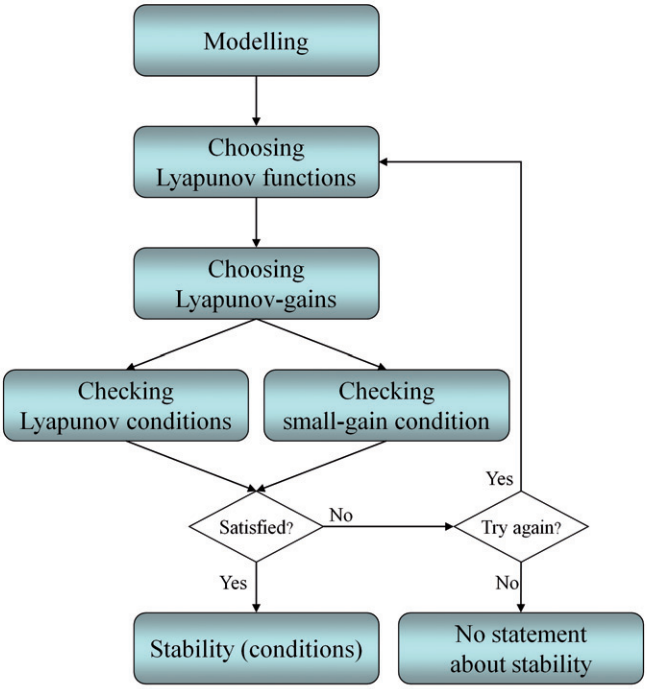

Figure 1. Flowchart of the procedure of a mathematical stability analysis.

and can be interpreted as (continuous) time. In this scenario the state $x_i(t)$ represents the WIP of subsystem i at time t.

Each plant of the network is represented by a complete shop floor scenario. It consists of three parallel production lines. Every line has three workstations and an input buffer in front of each workstation. The structure allows the parts to switch lines at every stage. The decision about changing the line is made by the part itself by internal control rules. These rules on the shop floor level are two different autonomous control methods: the queue length estimator (QLE) and the pheromone based (PHE) method.

The QLE enables parts to choose a workstation according to local information about their current workload. The parts are able to interact with others and to gather information about the current workload. Similar to the join-the-shortest-queue policy (Sharifnia 1997, Foley and McDonald 2001) a part collects information about the amount of waiting parts in the relevant buffers. Additionally, the parts calculate the waiting time for each alternative. Parts, using this method, will choose the workstation with the lowest workload to reduce their own throughput time (for a detailed description see Scholz-Reiter *et al*. 2005b).

The PHE method is the second autonomous control method. It is inspired by the behaviour of ants marking possible routes to food sources with pheromone trails. Succeeding ants are able to detect those trails. They will follow the trail with the highest concentration of pheromones (Parunak 1997). This process is transferred to the shop floor level of this scenario as follows. Parts save information about their waiting and processing time at a machine. Succeeding parts, entering a stage of the shop floor, compare these artificial pheromones by computing the average throughput time (TPT) data of the last

Table 1. Weighted adjacency matrix of the production network in factions (as [percentages]).

		To							
		Source	Sink	P1	P2	P3	P4	P5	P6
From	Source			100					
	Sink								
	P1				50	50			
	P2						60	40	
	P3						60	40	
	P4								100
	P5								100
	P6		50	50					

five parts and choose a line. A moving average of the TPT is used to model the evaporation process of natural pheromones. Similar approaches for modelling pheromone based autonomous control methods can be found in Peeters *et al.* (2001), and Armbruster *et al.* (2006).

We suppose that the production rate of all machines on the shop floor level and therefore all plants are autonomously controlled. This means that each machine and plant, respectively, has the ability to adjust its production rate up to some maximal capacity α_{ij} or α_i, respectively, $i = 1, \dots, 6$, $j = 1, \dots, 9$, depending on the current situation (on the current state of this machine or plant). For example, this can be achieved by varying work times of the workers or the number of used machines for the production.

Plant 1 obtains raw material from an external source, which is denoted by $u(t)$ and some material from plant 6. The raw material will be processed in plant 1 and 50% of the production output will be transported to plant 2 and 50% to plant 3, which can be interpreted as input of raw material to plants 2 and 3, respectively. Again the raw material will be processed and 60% of the output of plants 2 and 3 will be transported to plant 4 and 40% of the output of plants 2 and 3 to plant 5. The processed material of plants 4 and 5 will be transported to plant 6. Fifty percent of the output of plant 6 will be transported to some customers and will leave the system, whereas 50% of the production will be transported back to plant 1. This can be interpreted as recycling of the waste produced in plant 6. The material flow between different plants can be described by a weighted adjacency matrix. This matrix can be found in Table 1.

5.2 *Macroscopic modelling by using a model, based on differential equations*

The macroscopic approach is the description and analysis from a mathematical point of view. The production network scenario is modelled by differential equations, which are presented in this section.

In this section we write *subsystem i* for the *i*th plant. All six subsystems form the production network, which we call simply (whole) system. The internal structure on the shop floor level of all subsystems is ignored and each subsystem is modelled as one production plant by means of differential equations. All subsystems are controlled

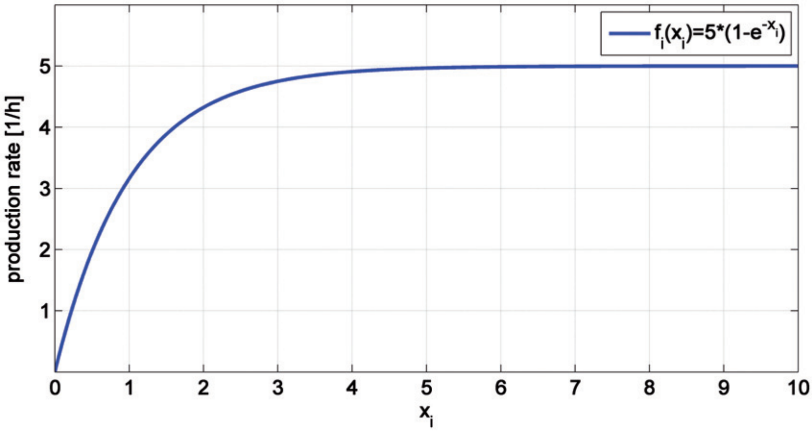

Figure 2. Production rate for $\alpha_i = 5$ depending on the state x_i.

autonomously by means of an autonomous adjustment of the production rates. For subsystem i the actual production rate is given by:

$$\tilde{f}_i(x_i(t)) := \alpha_i(1 - \exp(-x_i(t))), \quad i = 1, \ldots, 6,$$

where the positive real value α_i is the (constant) maximal production rate of subsystem i. An example with $\alpha_i = 5$ is displayed in Figure 2. $\tilde{f}_i$ converges to α_i, if the state of subsystem i is large and $\tilde{f}_i$ tends to zero, if the state of the ith subsystem tends to zero. Accordingly, a huge influx of raw material causes an increase of the production rate close to the maximum, whereas less influx of raw material leads to a production rate, which is almost zero.

When modelling the system by differential equations we assume that the processed material will be instantly transported and arrives at the succeeding subsystem at the same time t. This makes the model simple and descriptive to illustrate the stability analysis. Thus, modelling a logistic network by differential equations is only an approximation of the real world; an exact description is given by the DES model. A differential equation describes the time rate of change and sums up the inflow and the outflow of a system at continuous time t, where the outflow is subtracted. In simple words this is the derivative of the state. It is denoted by $\dot{x}_i(t)$.

With these considerations we model the given network by differential equations as follows:

$$\begin{aligned}
\dot{x}_1(t) &:= u(t) + \frac{1}{2}\tilde{f}_6(x_6(t)) - \tilde{f}_1(x_1(t)) \\
\dot{x}_2(t) &:= \frac{1}{2}\tilde{f}_1(x_1(t)) - \tilde{f}_2(x_2(t)) \\
\dot{x}_3(t) &:= \frac{1}{2}\tilde{f}_1(x_1(t)) - \tilde{f}_3(x_3(t)) \\
\dot{x}_4(t) &:= \frac{3}{5}\tilde{f}_2(x_2(t)) + \frac{3}{5}\tilde{f}_3(x_3(t)) - \tilde{f}_4(x_4(t)) \\
\dot{x}_5(t) &:= \frac{2}{5}\tilde{f}_2(x_2(t)) + \frac{2}{5}\tilde{f}_3(x_3(t)) - \tilde{f}_5(x_5(t)) \\
\dot{x}_6(t) &:= \tilde{f}_4(x_4(t)) + \tilde{f}_5(x_5(t)) - \tilde{f}_6(x_6(t)),
\end{aligned} \tag{4}$$

where $t \in \mathbb{R}_+$ is the continuous time, $x_i(t) \in \mathbb{R}_+$ is the WIP of a subsystem, $u(t) \in \mathbb{R}_+$ is the input of the first subsystem and $\tilde{f}_i(x_i(t))$ are the production rates of the subsystems (see Figure 2).

For this model we perform a stability analysis in the next section, using the tools introduced in Section 4. In the context of mathematical modelling by differential equations some remarks can be stated. The state of a subsystem may also represent other relevant parameters of the system, e.g., the number of unsatisfied orders. Furthermore, one can extend or change the given production network to describe any other scenario that can be more large and complex. It is possible to perform a stability analysis for the extended system, similar to the flowchart presented in Figure 1. Even more complex models can be analysed with this scheme and parameters can be determined, which guarantee stability.

To keep the stability analysis relatively simple, in order to introduce and explain the methods used in a comprehensible manner, we consider the production network described above.

5.3 *Microscopic modelling using a discrete event simulation model*

The microscopic approach is based on a DES model, which represents a more detailed view on this production network scenario. Due to the discrete nature of this model type and the disaggregation of all plants, some parameters have to be adjusted to provide comparability to the macroscopic model.

The DES model represents the material flow by discrete entities (parts) running through the network in contrast to the mathematical model, which is based on continuous equations. These parts arrive at plant P1 in certain time intervals, which are determined by cumulating the arrival rate $u(t)$. Whenever the cumulated arrival rate $u(t)$ reaches an integer value a part enters the system at the corresponding time point t. Accordingly, the incoming workload in the DES model is equivalent to the incoming workload in the mathematical model.

The internal structure on the shop floor level with the QLE and PHE method of each plant, described in Section 5.1, is modelled. Due to the three parallel production lines, each line offers one third of the maximal production rate α_i of the whole plant, such that the parameter of the maximal production rate of each workstation α_{ij}, $i = 1, \ldots, 6$, $j = 1, \ldots, 9$, is chosen to be $\alpha_{ij} = \alpha_i/3$.

An illustration of the production network and the macroscopic and microscopic view is given in Figure 3.

In the following section the stability analysis is done by applying a macroscopic and a microscopic view on the network. The differential equation model is used to determine parameters for which the stability of the given scenario is guaranteed on the network level (macroscopic view). These bounds of parameters are interpreted as follows: for values of parameters above this bound stability can be guaranteed mathematically, however below this bound stability cannot be guaranteed. Furthermore, the mathematical model is simulated by the software Matlab to verify and to improve the calculated bound of stability numerically.

The results of the stability analysis of the macroscopic view will be compared with the results of the microscopic view, in order to obtain a possibly precise stability region similar to Scholz-Reiter *et al.* (2005a). To exclude that possible deviations of the stability results of the mathematical model and the DES model are caused by modelling errors, the macroscopic Matlab simulation will also be compared with the results of the DES model. Since both models, the mathematical and the DES models, describe the same scenario, but on different abstraction levels (macroscopic and microscopic view), the results can be compared, where the results of the microscopic view are more detailed.

6. Mathematical stability analysis of the given production network

In this section we perform the stability analysis, described in Section 4, for the given production network, modelled by a macroscopic view by differential equations and derive stability conditions.

Consider the network given in Figure 3 and modelled in Section 5 by a macroscopic view, in particular Equations (4). The question arises, under which conditions for the maximal production rates α_i the subsystems are ISS or LISS, respectively. This means that the states of all subsystems will not increase to infinity. In other words we are looking for the smallest α_i, such that all subsystems are stable.

To answer this question we follow the steps of the mathematical stability theory presented in the flowchart in Figure 1: we choose $V_i(x_i) = x_i$, $i = 1, \ldots, 6$ as the Lyapunov

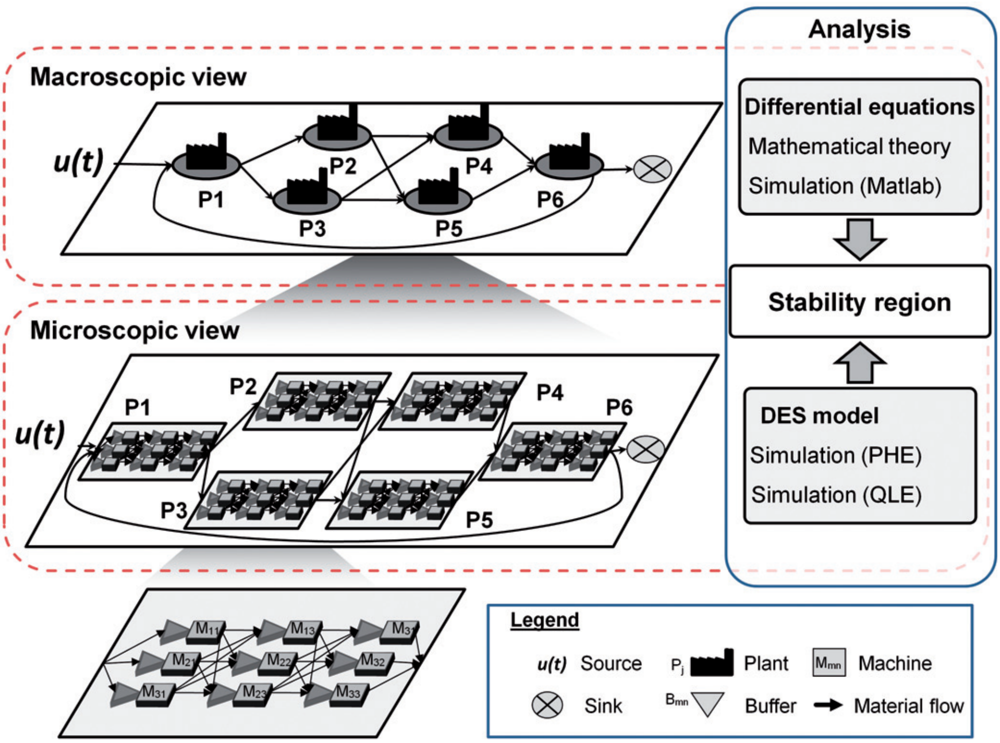

Figure 3. Production network scenario and both modelling views.

function candidates for the subsystems, define the Lyapunov gains by:

$$\chi_u(u(t)) := -\ln\left(1 - \frac{u(t)(\|u\|_\infty + 0.5\cdot\alpha_6)}{\|u\|_\infty(1-\varepsilon_u)\alpha_1}\right), \quad 1 > \varepsilon_u > 0$$

$$\chi_{61}(x_6) := -\ln\left(1 - \frac{\|u\|_\infty + 0.5\cdot\alpha_6}{(1-\varepsilon_{61})\alpha_1}(1-\exp(-x_6))\right), \quad 1 > \varepsilon_{61} > 0$$

$$\chi_{1j}(x_1) := -\ln\left(1 - \frac{0.5\cdot\alpha_1}{(1-\varepsilon_{1j})\alpha_j}(1-\exp(-x_1))\right), \quad 1 > \varepsilon_{1j} > 0,\ j = 2,3$$

$$\chi_{j4}(x_j) := -\ln\left(1 - \frac{1.2\cdot\alpha_2}{(1-\varepsilon_{j4})\alpha_4}(1-\exp(-x_j))\right), \quad 1 > \varepsilon_{j4} > 0,\ j = 2,3$$

$$\chi_{j5}(x_j) := -\ln\left(1 - \frac{0.8\cdot\alpha_2}{(1-\varepsilon_{j5})\alpha_5}(1-\exp(-x_j))\right), \quad 1 > \varepsilon_{j5} > 0,\ j = 2,3$$

$$\chi_{j6}(x_j) := -\ln\left(1 - \frac{\alpha_4+\alpha_5}{(1-\varepsilon_{j6})\alpha_6}(1-\exp(-x_j))\right), \quad 1 > \varepsilon_{j6} > 0,\ j = 4,5,$$

and show that $V_i(x_i)$ are the ISS or LISS Lyapunov functions of the subsystems. Therefore the two conditions of an ISS or LISS Lyapunov function, respectively, have to be verified. Further details on the choice of the gains and this verification can be found in Dashkovskiy *et al.* (2009), which can be used here in a similar way, so we conclude that $V_i(x_i)$ are the ISS or LISS Lyapunov functions of the subsystems. This means that all subsystems are ISS or LISS, respectively.

In this case the gain-matrix is of the form:

$$\Gamma = \begin{pmatrix} 0 & \chi_{12} & \chi_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & \chi_{24} & \chi_{25} & 0 \\ 0 & 0 & 0 & \chi_{34} & \chi_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & \chi_{46} \\ 0 & 0 & 0 & 0 & 0 & \chi_{56} \\ \chi_{61} & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

and satisfies the small-gain condition (see Dashkovskiy *et al.* (2009) for calculation details). The gain-matrix contains information about the interconnections of a system: every entry (gain) represents an interconnection. To guarantee that the Lyapunov gains are well-defined we obtain conditions for α_i:

$$\begin{aligned} \alpha_1 - 0.5\cdot\alpha_6 > \|u\|_\infty \Rightarrow\ & \alpha_1 > \|u\|_\infty + 0.5\cdot\alpha_6, \\ & \alpha_2 > 0.5\cdot\alpha_1, \\ & \alpha_3 = \alpha_2 > 0.5\cdot\alpha_1, \\ & \alpha_4 > 1.2\cdot\alpha_2, \\ & \alpha_5 > 0.8\cdot\alpha_2, \\ & \alpha_6 > \alpha_5 + \alpha_4, \end{aligned}$$

from which we obtain that subsystem 1 is LISS with $\rho_1^u := \alpha_1 - 0.5\cdot\alpha_6 > \|u\|_\infty$. Note, that these conditions are derived only for the particular scenario. For other scenarios one may obtain other stability conditions. Subsystems 2 to 5 are ISS. If these conditions are satisfied for any input $u(t)$, then by Theorem 1 in Section 4 for LISS we obtain that the whole system is LISS, i.e., the states of the subsystems and therefore the state of the whole system is bounded. Note that the choice of the input $u(t)$ is arbitrary. The higher $\|u\|_\infty$, the higher the maximal production rates to guarantee the fulfilment of the conditions for α_i and to guarantee stability.

The stability conditions form a system of linear inequalities. By solving this system for given input $u(t)$ we can calculate explicit values for α_i. The smallest values solving the system of linear inequalities are the α_i we are looking for. For them and larger values stability of the system is guaranteed and the smallest α_i form a bound of stability for given input $u(t)$.

The bounds calculated by the help of LISS Lyapunov functions are sufficient for the stability of the system. These are worst case bounds for the case we have no information about the input u. However, in practice we know about seasonal changes and other specific variation properties of u. This information can be used to enlarge the set of parameters so that the whole system remains stable, i.e., to guarantee stability for some smaller values of the α_i. For these smaller production rates, from a mathematical point of view, stable systems behaviour neither can be guaranteed nor negated. Thus, the use of simulation models may help to refine the calculated stability results. In this respect the DES model is used to analyse the obtained stability regions (bounds for α_i) for a given set of inputs $u(t)$.

In order to obtain refined bounds of stability we simulate the system by the DES model, where the results of the mathematical stability analysis are used as starting points. This approach allows determining stability parameters of a complex autonomously controlled production network with less time consumption compared to a pure trial and error simulation study. The macroscopic and microscopic views complement one another.

Furthermore, to verify the comparability between the mathematical model and the DES model we simulate the system modelled by differential equations with Matlab. We also obtain refined bounds of stability in comparison to the calculated ones. These results are displayed in Figure 6. Note that they can only be verified by simulations and not by mathematical theory.

7. Simulation results

In this section the results of the stability analysis of the macroscopic and the microscopic view for varying input parameters of demand are presented and refined by simulations.

For the simulations we choose the following input $u(t)$:

$$u(t) := AV \cdot (\sin(t) + 1) + 5,$$

where AV is a real positive number. The choice of u takes seasonal changes of the demand into account and gives some dynamics, which may appear in the real world. This choice is one possibility for representing a demand. For other choices of the input one obtains different stability regions.

Figure 4 shows the arrival rate for different arrival rate amplitude variations ($AV = 5$, $AV = 6$ and $AV = 7$). Variations of AV cause a change of the mean arrival rate, as well as a change of the amplitude of $u(t)$. The sinusoidal arrival rate $u(t)$ and the factor AV are given. Furthermore, AV is varied in order to analyse different demand situations.

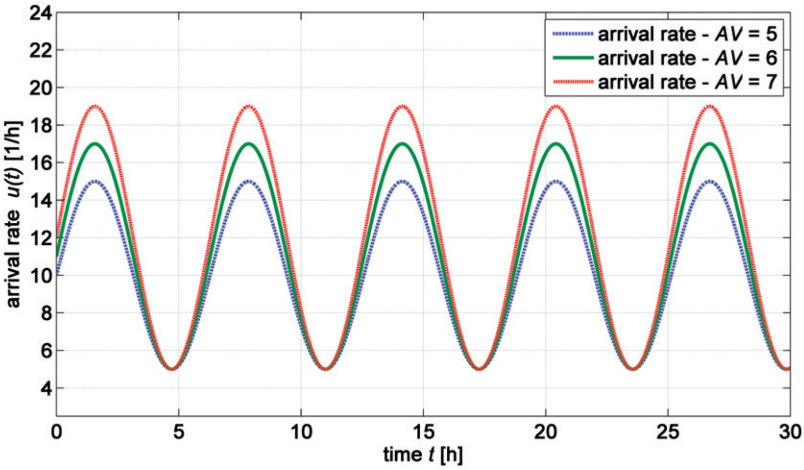

Figure 4. Varied arrival rates for $AV = 5$, $AV = 6$, $AV = 7$.

The variation of the arrival rate amplitude AV is increased stepwise in different simulations from $AV = 1$ to $AV = 80$. Note, that AV determines the amplitude of the arrival rate $u(t)$, according to Figure 4.

Now we can calculate for each AV explicit α_i, which satisfy the stability conditions. For example, let $AV = 10$, then $\|u\|_\infty = 25$ and the production rates have to be chosen as $\alpha_1 > 50$, $\alpha_2 > 25$, $\alpha_3 > 25$, $\alpha_4 > 30$, $\alpha_5 > 20$, $\alpha_6 > 50$ to guarantee stability by mathematical theory. If AV is varied from 1 to 80 we obtain a stability region.

In order to find the smallest α_i for the DES model for certain values of AV, the calculated smallest α_i of the mathematical model are used. These production rates are reduced in steps of 1% per plant until the simulated WIP starts to grow persistently in a time interval of 30 hours to about 10%. In this case the simulation is stopped, since it is reasonable to assume that the WIP of a plant is no more bounded over time. The rise of 10% is sufficient, because it was observed that values from 1% to 9% provide nearly similar results. Values above an average increase of 20% yield to high errors in the decision if a system is stable or not.

Figure 5 shows exemplary stable and unstable simulation results of the Matlab simulation and of the DES simulation with the QLE method. If the arrival rate amplitude is set to $AV = 10$ and the production rates were chosen as $\alpha_1 = 30.1$, $\alpha_2 = 15.1$, $\alpha_3 = 15.1$, $\alpha_4 = 18.2$, $\alpha_5 = 12.2$, $\alpha_6 = 30.5$, then the stable situation is observed.

Choosing the production rates for the plants only a bit smaller ($\alpha_1 = 29$, $\alpha_2 = 14.51$, $\alpha_3 = 14.51$, $\alpha_4 = 17.52$, $\alpha_5 = 11.52$, $\alpha_6 = 29.5$) the behaviour becomes unstable as shown in Figure 5.

In the unstable situation the WIP of the plants P1 and P5 does not remain bounded and grows continuously with an average of 0.43 and 0.081 units per time unit. A simulation showing this kind of dynamical behaviour is stopped and evaluated as unstable. On the other hand the WIP in the stable situation exhibits variations, due to the fluctuating arrival rate, but it does not grow continuously and remains bounded for all times.

Figure 6 presents the smallest maximal production rate of plant P5 from the microscopic and the macroscopic view, representatively. Figures of other plants are not displayed here, because they are similar to Figure 6, unlike some quantitative information.

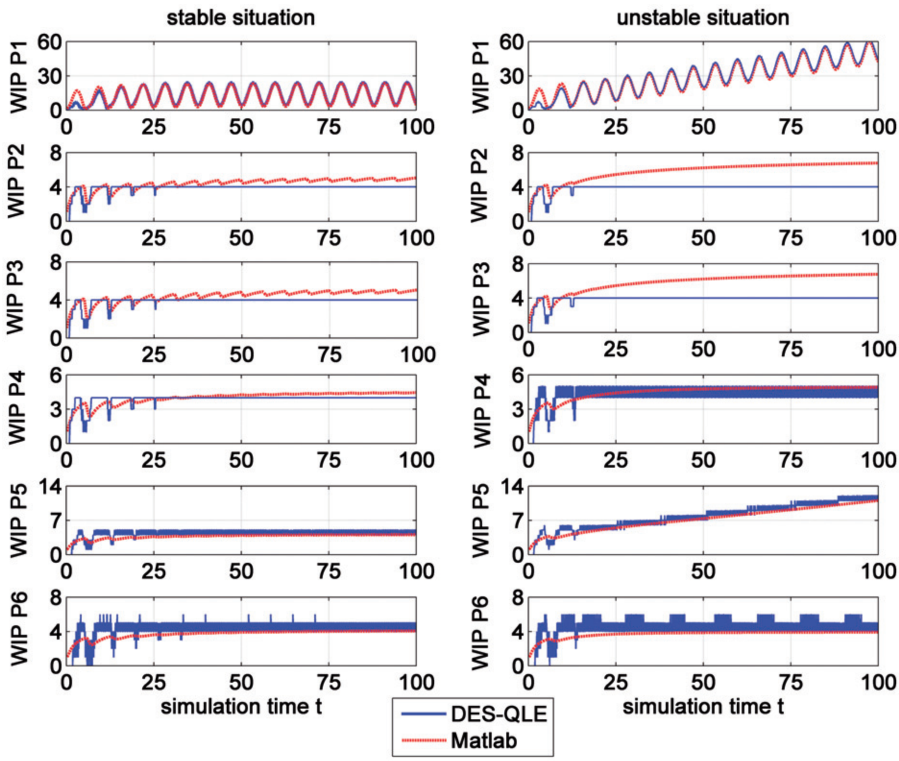

Figure 5. Stable and unstable behaviour of the Matlab and the DES simulation.

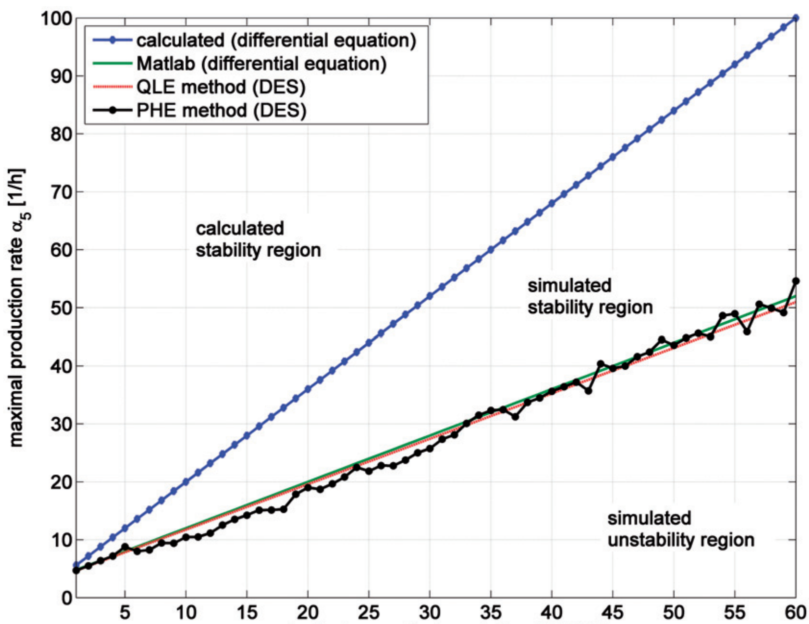

Figure 6. Bounds of stability of different models and methods.

Regarding the calculated smallest α_i it can be observed that the smallest α_i grows linear with the AV. This can be explained by an increased incoming workload for higher values of the AV. Due to the increase of the mean arrival rate the plants have to offer more capacity to process the higher workload. Thus, the smallest α_i have to be larger.

From a mathematical point of view stability of the system can only be guaranteed for production rates on and above the mathematically calculated curve in Figure 6, whereas below this curve we cannot give any information about stability of the system. Here, simulations were used to find more precise bounds of stability. For the maximal production rates above the curve in Figure 6, obtained by simulations of the DES model, stability was observed, whereas below this curve the behaviour becomes unstable. The region between the curve (the bound of stability) found by the mathematical stability analysis and the bounds of the simulations is the more precise identification of the stability region (denoted by *simulated stability region* in Figure 6). It should be pointed out, that for different scenarios one obtains different stability regions.

Regarding the smallest α_i, obtained by the DES model, it can be noticed that both autonomous control methods provide nearly the same results. The curves of the smallest α_i, obtained by the application of the QLE and the PHE method, almost overlap for each plant. A nearly linear trend of the smallest α_i for rising values of AV can be found for both methods. However, the curve of the PHE method behaves more back and forth than the curve of the QLE method. This may be caused by the usage of data from the past for parts using the PHE method. Due to this past information it may happen that single decisions of parts are not adequate and lead to longer waiting times and consequently to a higher WIP. Thus the bound of stability of the PHE method is not as sharp as the bound of the QLE method. Summarising the results of the DES model one can say that the different autonomous control methods cause only slight differences in view of the stability regions.

The simulation time of the DES model depends on the number of parts, which are in the network: the higher the AV, the larger is the number of parts. Using the QLE method the time needed for one single simulation at the time interval from 0 to 100 is 13.48 seconds for $AV = 5$ and 158.38 seconds for $AV = 100$, where a computer with a 3 GHz processor and 4 GB RAM was used.

Figure 6 also contains the results of the Matlab simulation. These simulations and the model used to calculate the smallest α_i are based on the same differential equations. Thus, the comparability of the simulated and calculated results, based on differential equations, is given. Figure 6 depicts that the results of the Matlab simulation also nearly overlap with the results of the DES model. Hence, the mathematical model is a good approximation of the scenario and the results of both views (macroscopic and microscopic) can be compared. The average differences between the QLE method and the Matlab simulation are 2.01% and between the Matlab simulation and the PHE method are 2.66%, respectively.

Regarding Figure 6 it can be noticed that compared to the mathematically calculated bounds of stability, all bounds of the simulations are lower and more precise for all values of AV. This result is caused by the usage of the worst case within the mathematical stability property ISS, namely the supremum norm $\|\cdot\|_\infty$. In particular for oscillating inputs (e.g., seasonal changes of the demand) the maximal value is used for all the time to derive stability parameters, such that lower inputs will not be considered over the time. Whereas in the simulation (DES model and Matlab) the actual input for time t is used, which is not the maximal value for all the time for an oscillating input and therefore lower stability parameters can be obtained for the simulation. As a result, the gap between calculated and simulated stable maximal production rates increases with the parameter AV.

For complex systems, especially for large-scale networks with feedback loops, the determination of the maximal production rates to keep the system stable is a challenging task. To identify the stability region by simulations one has to check the stability of the network for a large set of maximal production rates. As presented above, the simulation time increases exponentially by the number of parts and plants in the network. In particular, for large-scale networks this is a problem, which cannot be solved in an acceptable time. The dual approach presented, helps to derive and refine stable system parameters in applicable time, which is the major advantage of this approach. It can be transferred to general networks, due to the mathematical stability theory, presented in Section 4, is for general networks (n coupled systems) and the simulations can also be adapted to larger and complex systems.

8. Conclusions and outlook

Summarising the results presented above one can say that the stability region of an autonomously controlled production network can be precisely obtained and verified in an acceptable time by a dual approach. An analytical (mathematical) investigation was used to derive conditions, which guarantee stability of the network for arbitrary input. By specification of the input, parameters for which stability of the system can be guaranteed were calculated. These results were refined by the usage of a DES model with more specific system parameters, i.e., additional autonomous control methods on the shop floor level. It was shown that more precise bounds of stability, which form the stability region, can be found by simulations compared to the mathematically calculated bounds of the mathematical stability analysis. A comparison of the results of the DES model and the Matlab simulations showed that the approximation by the use of the mathematical model is suitable for this scenario.

A mathematical model, based on differential equations, can be used to calculate parameters, for which stability can be guaranteed. These parameters are sufficient for stability of the system and they form bounds of stability for certain production networks. Using these parameters the bounds of stability can be refined by a simulation model (DES). This approach allows determining stability parameters of a complex autonomously controlled production network with less time consumption compared to a pure trial and error simulation study. The mathematical stability analysis and simulations by the DES model complement one another.

This approach can be transferred to other more complex logistic networks. By application of the stability analysis as presented here one can derive stability parameters to guarantee stability of the more complex networks. The parameters help to design the network to avoid negative outcomes and to achieve logistic and economic goals.

In future research activities we will consider transportation connections (delivery times) between the plants to display the reality in our models in more detail. Mathematically we have to take into account time-delay, hybrid and/or switched systems and its stability theory, which is an actual mathematical research topic, especially for interconnected dynamical systems. On the other hand the QLE and PHE method have to be adapted to take into account transportation and in particular to handle perturbations in production networks (e.g., closed transportation connections, transportation delays).

Also, the autonomous control methods have to be described mathematically and to be integrated in the differential equations. Then, it is possible to perform a stability analysis on these models, which are closer to the reality.

By taking into account transportation connections (delivery times) and stochastic inputs (demands) a further dynamical effect may occur, known as the 'bullwhip effect'. This effect increases the amplitude of the WIP of a system and the throughput time and causes surplus production and high inventory costs. The investigation and analysis of the influence of the application of the models and the autonomous control methods presented in this paper to reduce the bullwhip effect will also be an interesting research topic in the future.

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