Local Capacity H_∞ Control for Production Networks of Autonomous Work Systems with Time-Varying Delays

H.R. Karimi, N.A. Duffie, S. Dashkovskiy

Abstract—**This paper considers the problem of local capacity H_∞ control for a class of production networks of autonomous work systems with time-varying delays in the capacity changes. The system under consideration is modelled in a discrete-time singular form. Attention is focused on the design of a controller gain for the local capacity adjustments which maintains the work in progress (WIP) in each work system in the vicinity of planned levels and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. In terms of a matrix inequality, a sufficient condition for the solvability of this problem is presented using an appropriate Lyapunov function, which depends on the size of the delay and is solved by existing convex optimization techniques. When this matrix inequality is feasible, the controller gain can be found by using LMI Toolbox Matlab. Finally, numerical results are provided to demonstrate the proposed approach.**

I. INTRODUCTION

Production networks are emerging as a new type of cooperation between and within companies, requiring new techniques and methods for their operation and management ([1]-[3]). Coordination of resource use is a key challenge in achieving short delivery times and delivery time reliability. These networks can exhibit unfavourable dynamic behaviour as individual organizations respond to variations in orders in the absence of sufficient communication and collaboration, leading to recommendations that supply chains should be globally rather than locally controlled and that information sharing should be extensive ([4]-[5]). The global control becomes difficult and vulnerable in case of large size and high complexity of production networks. This is due to permanent changes of, e.g., market requirements, order sizes and internal disturbances such as machine failures, communication delays, information losses etc. This can destabilize the dynamics of a production network and lead to low performance and economic losses. A compromise is to allow some entities of a network, e.g., single machines or separate plants to make decisions by their own based on local situation and available information. Such entities are called autonomous work systems in this paper. A set of rules to make decisions for a single autonomous work system is called autonomous control. However, the dynamic and structural complexity of these emerging networks inhibits collection of the information necessary for centralized planning and control, and decentralized coordination must be provided by logistic processes with autonomous capabilities [6]. Furthermore, to develop and analyze autonomous control strategies dynamic models are required. For that different modeling approaches are investigated regarding their abilities to describe an exemplary scenario—an autonomously controlled production network. A discrete-event simulation model is compared to a deterministic fluid model for a continuous product queue, both based on previous work in [7]-[8]. Recently, models and control strategies based on the idea of pheromones was developed in [9]. That is, the decision which path to choose through the production network is not made by a manager or operator, but by the individual part itself, based on the 'experience' of other parts of the same type.

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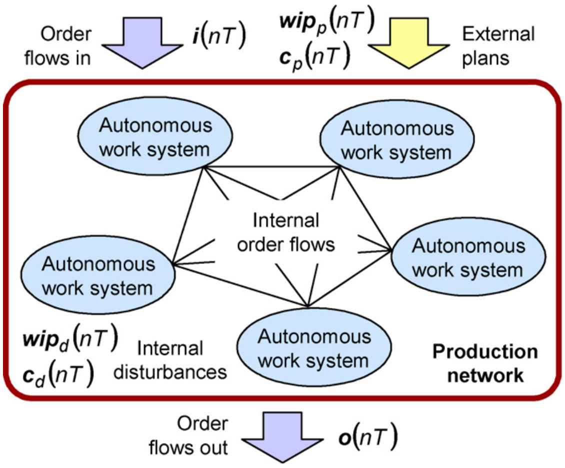

Fig. 1. Production network consisting of a group of autonomous work systems.

A production network with several autonomous work systems is depicted in Fig. 1. The behaviour of such a network is affected by external and internal order flows, planning, internal disturbances, and the control laws used locally in the work systems to adjust resources for processing orders. In prior work, sharing of capacity information between work systems has been modelled along with the benefits of alternative control laws and reducing delay in capacity changes ([10]-[12]). Several authors have described both linear

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and nonlinear dynamical models for control of variables such as inventory levels and work in progress (WIP), including the use of pipeline flow concepts to represent lead times and production delays ([2], [13]-[14]). Delivery reliability and delivery time have established themselves as equivalent buying criteria alongside product quality and price [1]. High delivery reliability and short delivery times for companies require for high schedule reliability and short throughput times in production. In order to manufacture economically under such conditions, it is necessary to minimise WIP levels in production and utilise operational resources in the best possible way ([3], [15]).

Production Planning and Control (PPC) has become more challenging as manufacturing companies adapt to a fast changing market ([16]). Current PPC methods often do not deal with unplanned orders and other types of turbulence in a satisfactory manner [17]. Assumptions such as infinite capacity and fixed lead time are often made, leading to a static view of the production system may not be valid because WIP affects lead time and performance, while capacity is finite and varies both according to plan and due to unplanned disturbances such as equipment breakdowns, worker illness, market changes etc. Understanding the dynamic nature of production systems requires new approaches for the design of PPC based on company's logistics [18]. The controllers implicitly interact to adjust capacity to eliminate backlog as the system maintains its planned WIP level [17]. A discrete closed-loop PPC model was developed and analyzed by Duffie and Falu [19] in which two discrete controllers, one for backlog and one for WIP, with different periods between adjustments of work input and capacity, respectively, were selected and evaluated using transfer function analysis and time-response simulation. A second architecture for continuous WIP control and discrete backlog control, with delay capacity adjustment, was developed and analyzed by Ratering and Duffie for cases of high and low WIP [20]. This analysis was facilitated by linearization of the logistic function using operating point analysis. A closed-loop production planning and control concept has been employed with adaptive inventory control in decision support systems in a multi-product medical supplies market [16]. State-space models have been used for switching between a library of optimal controllers to adjust WIP in serial production systems in the presence of machine failures [15], and switching of control policies in response to market strategies has been investigated in [21].

On the other hand, delay systems represent a class of infinite-dimensional systems largely used to describe propagation and transport phenomena or population dynamics [22]. Delay differential systems are assuming an increasingly important role in many disciplines like economic, mathematics, science, and engineering. The delay effects problem on the stability of systems including delays in the state and/or input is a problem of recurring interest since the delay presence may induce complex and undesired behaviors for the schemes ([23]-[26]). For the continuous-time case, most results have been obtained based on the modified Riccati equation/inequality approach [27] and the linear matrix inequality (LMI) approach ([23]-[26], [28]). It should be pointed out that, the discrete-time systems with time-delay have received little attention compared with its continuous-time counterpart ([29]-[33]). The main reason for this is that for precisely known discrete-time systems with constant time-delay, it is always possible to obtain an augmented system without delayed states [34]. This approach, however, does not seem to be suitable for time-varying delay, delay-independent stability characterization, and for robust-system stabilization [35]. With regard to the stability analysis issue, Verriest and Ivanov in [36] studied the sufficient conditions for the asymptotic stability of the discrete-time state delayed systems by using an algebraic matrix inequality approach. Concerning the problem of designing control systems, Song and Kim in [37] have established the H_∞ control problem for linear discrete-time uncertain time-delay systems and a sufficient condition has been derived in terms of a Riccati-like matrix inequality. In the context of discrete time-delay systems, sufficient conditions for the solvability of the H_∞ control problem was obtained in [38] in terms of a modified Riccati equation. Recently, the problem of robust H_∞ control for a class of discrete systems with time-varying delays and time-varying norm-bounded parameter uncertainties was studied in [39].

In this paper, we contribute to the further development of a local capacity control design for a class of production networks of autonomous work systems with time-varying delays in the capacity changes. The system under consideration is modelled as a discrete-time singular form. An appropriate Lyapunov function is constructed in order to establish a delay-range-dependent sufficient condition in terms of a matrix inequality for finding a controller gain for the local capacity adjustments which maintains the WIP in each work system in the vicinity of planned levels and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. When this matrix inequality is feasible, the controller gain can be found by using LMI Toolbox Matlab. Finally, numerical results are provided to demonstrate the proposed approach.

Notations. The superscript 'T' stands for matrix transposition; $\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{n\times m}$ is the set of all real m by n matrices. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. $col\{\cdots\}$ and $diag\{\cdots\}$ represent, respectively, a column vector and a block diagonal matrix and the operator $sym(A)$ represents $A+A^T$. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote, respectively, the smallest and largest eigenvalue of the square matrix A. The symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. If $\Sigma\in\Re^{m\times n}$ and $rank(\Sigma)=r$, the orthogonal complement $\Sigma^\perp$ is defined as a possibly nonunique $n\times(n-r)$ matrix with rank $n-r$, such that $\Sigma\Sigma^\perp=0$.

II. MODEL OF AUTONOMOUS WORK SYSTEMS

Assume that there are N work systems in a production network, as shown in Figure 1, and that vector $\boldsymbol{i}(nT)$ is the rate at which orders are input to the N work systems from sources external to the production network, which is constant over time $nT \le t < (n+1)T$ where $n = 0,1,2,\cdots$ and T is a time period between capacity adjustments. The total orders that have been input to the work systems up to time $(k+1)T$ then can be represented as the vector [6]

$$w_j((n+1)T) = w_j(nT) + T(i(nT) + R^T c_a(nT)) \quad (1a)$$

$$w_o((n+1)T) = w_o(nT) + Tc_a(nT) \quad (1b)$$

$$wip_a(nT) = w_j(nT) - w_o(nT) + w_d(nT) \quad (1c)$$

where $j = 1,2,\cdots,N$ and the elements of vector $i(nT)$, $n = 0,1,2,\cdots$, represent the rate at which orders flow from external sources into N work systems at discrete instants in time separated by time interval T. Eq. (1a) consists of the total number of orders that have been input to each work system where $c_a(nT)$ is the actual capacity of each work system and R is a matrix in which element approximates the fraction of the flow out of work system j that flows into work system k. Eq. (1b) presents the total number of orders that have been output by each work system and the WIP is formulated in Equation (1c) where $w_d(nT)$ represents local work disturbance, such as rush order, that affect the work system. Furthermore, the actual capacity of each work system depends on three components as follows:

$$c_a(nT) = c_p(nT) + c_m((n-d(n))T) - c_d(nT) \quad (1d)$$

where $c_d(nT)$ represents local capacity disturbances such as equipment failures, $c_p(nT)$ denotes planned capacities of the work systems and $c_m(nT)$ represents local capacity adjustments to maintain the WIP in each work system in the vicinity of the planned levels $wip_p(nT)$ using straightforward proportionality k_c and is described in the form of

$$c_m(nT) = k_c(wip_a(nT) - wip_p(nT)) \quad (1e)$$

It is assumed that a time-varying delay $d(n)T$ exists in the capacity changes $c_m(nT)$ and satisfies

$$d_1 \le d(n) \le d_2 \quad (2)$$

and the planned capacity and WIP are also assumed to be known and delay free in advance.

Information used in calculating full capacity is assumed to be available $d(n)T$ time periods in advance regardless of whether it is the result of external planning or derived from information shared within the network.

Remark 1. It is clear from Equations (1) that the fundamental dynamic properties of the network are a function of order-flow structure. With the objective of establishing and maintaining consistent and desirable fundamental dynamic properties, we may consider a network in which each work system shares expected capacity information with all other work systems in the network, allowing individual work systems to locally compensate for physical order-flow coupling.

Remark 2. Capacity adjustments can be large, and there can be relatively large differences between successive capacity adjustments. Such adjustments must be acceptable in application. Delay in capacity adjustment has been included to represent the inability to make instantaneous adjustments. For this case, $d(n)$ is called an interval-like or range-like time-varying delay [25], [39]. It is also noted that this kind of time-delay describes the real situation in many practical engineering systems. For example, in the filed of networked control systems, the network transmission induced delays (either from the sensor to the controller or from the controller to the plant) can be assumed to satisfy (2) without loss of generality [40].

Eqs. (1a)-(1e) can be combined to obtain a discrete-time singular model for the system:

$$E\,X((n+1)T) = A\,X(nT) + B\,H\,X((n-d(n))T) + C\,W(nT) \quad (3a)$$

$$Z(nT) = L_1\,X(nT) + L_2 H\,X((n-d(n))T) + L_3\,W(nT) \quad (3b)$$

where $X(nT) := [w_j(nT)^T, w_o(nT)^T, c_m(nT)^T]^T$, $W(nT) :=$ $[i(nT)^T, wip_p(nT)^T, c_p(nT)^T, w_d(nT)^T, c_d(nT)^T]^T$, $A = A_0 + k_c A_1$ and $C = C_1 + k_c C_2$ with

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix},\; A_0 = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -I \end{bmatrix},\; A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & -I & 0 \end{bmatrix},\; B = \begin{bmatrix} R^T \\ I \\ 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}^T,\; C_1 = \begin{bmatrix} I & 0 & R^T & 0 & -R^T \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\; C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -I & 0 & I & 0 \end{bmatrix},$$

where $Z(nT)$ denotes a controlled output of the system and the matrices L_1, L_2, L_3 are defined in Section 4.

Then the H_∞ control problem to be addressed in this paper can be formulated as finding the controller gain k_c such that

i) The system (3a) is asymptotically stable when $W(nT) = 0$.

ii) Under the zero-initial condition and for any nonzero $W(nT) \in L_2$, the controlled output $Z(nT)$ satisfies the H_∞ performance measure,

$$\|Z(nT)\|_2 < \gamma \|W(nT)\|_2, \quad (4)$$

where γ is a prescribed scalar.

III. MAIN RESULTS

In this section, sufficient conditions for the solvability of the local capacity H_∞ control problem are proposed using the Lyapunov method and an LMI approach is developed.

3.1 Stability analysis

In this section, assuming that the control gain k_c is known, new delay-range-dependent sufficient conditions for the local capacity H_∞ control problem formulated in the previous section are presented.

Theorem 1. For given scalars $d_1, d_2 > 0$, the system (3) is asymptotically stable and satisfies the H_∞ performance bound γ by the control gain k_c, if there exist some matrices

N_1, N_2, N_3 and positive-definite matrices P and Q such that the following matrix inequality is feasible,

$$\begin{bmatrix} \Pi_{11} & T_1^T N_2^T & N_1 T_2 + T_1^T N_3^T & A^T P & L_1^T \\ * & -Q & N_2 T_2 & B^T P & L_2^T \\ * & * & sym\{N_3 T_2\} - \gamma^2 I & C^T P & L_3^T \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{5}$$

with $d_{12} = d_2 - d_1$ and $\Pi_{11} = -E^T PE + (d_{12}+1) H^T QH + sym\{N_1 T_1\}$, $T_1 := [k_c I, -k_c I, -I]$ and $T_2 := [0, -k_c I, 0, k_c I, 0]$.

Proof: Consider the Lyapunov function candidate in the following form

$$V(nT) = X(nT)^T E^T P E\, X(nT) + V_1(nT) + V_2(nT) \tag{6}$$

where

$$V_1(nT) = \sum_{i=n-d(n)}^{n-1} X(iT)^T H^T Q H\, X(iT) \tag{7}$$

$$V_2(nT) = \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=n+j-1}^{n-1} X(lT)^T H^T Q H\, X(lT)\,. \tag{8}$$

Then

$$\begin{aligned} &V_1((n+1)T) - V_1(nT) \\ &= \sum_{i=n+1-d(n+1)}^{n} X(iT)^T H^T Q H\, X(iT) - \sum_{i=n-d(n)}^{n-1} X(iT)^T H^T Q H\, X(iT) \\ &\le \sum_{i=n+1-d(n+1)}^{n-d_1} X(iT)^T H^T Q H\, X(iT) + X(nT)^T H^T Q H\, X(nT) \\ &\quad - X((n-d(n))T)^T H^T Q H\, X((n-d(n))T) \end{aligned} \tag{9}$$

Now, by some calculation, we derive

$$\begin{aligned} &V_2((n+1)T) - V_2(nT) \\ &= d_{12} X(nT)^T H^T Q H\, X(nT) - \sum_{j=n+1-d_2}^{n-d_1} X(iT)^T H^T Q H\, X(iT) \end{aligned} \tag{10}$$

From (2) and (7)-(10), it is easy to see that

$$\begin{aligned} &V_1((n+1)T) + V_2((n+1)T) - V_1(nT) - V_2(nT) \\ &\le (d_{12}+1) X(nT)^T H^T Q H\, X(nT) \\ &\quad - X((n-d(n))T)^T H^T Q H\, X((n-d(n))T) \end{aligned} \tag{11}$$

Using the obtained inequalties (10)-(11), the following result is obtained

$$\begin{aligned} &V((n+1)T) - V(nT) \\ &\le (A X(nT) + B H X((n-d(n))T) + C W(nT))^T P \\ &\quad \times (A X(nT) + B H X((n-d(n))T) + C W(nT)) \\ &\quad - X(nT)^T E^T P E\, X(nT) + (d_{12}+1) X(nT)^T H^T \\ &\quad \times Q H\, X(nT) - X((n-d(n))T)^T H^T Q H\, X((n-d(n))T) \end{aligned} \tag{12}$$

Moreover, from (1), the following equation holds for any matrices N_1, N_2, N_3 with appropriate dimensions:

$$\begin{aligned} &(X(nT)^T N_1 + c_m((n-d(n))T)^T N_2 + W(nT)^T N_3) \\ &\qquad \times (T_1 X(nT) + T_2 W(nT)) = 0 \end{aligned} \tag{13}$$

Furthermore, in the case of $W(nT) = 0$ it follows from (12) that

$$V((n+1)T) - V(nT) \le \hat{\xi}(nT)^T \hat{\Pi} \hat{\xi}(nT) \le -\lambda_{\min}(-\hat{\Pi}) \left|\hat{\xi}(nT)\right|^2 \tag{14}$$

where $\hat{\xi}(nT) = [X(nT)^T, c_m((n-d(n))T)^T]^T$ and

$$\hat{\Pi} = \begin{bmatrix} \Pi_{11} + A^T PA & A^T PB \\ * & B^T PB - Q \end{bmatrix}. \tag{15}$$

On the other hand, considering the Lyapunov function (6), one gets

$$\lambda_{\min}(E^T PE) \left|X(nT)\right|^2 \le V(nT) \le \alpha_1 \left|X(nT)\right|^2 + \alpha_1 (d_{12}+1) \sum_{l=n-d_2}^{n-1} \left|X(lT)\right|^2 \tag{16}$$

where $\alpha_1 = \max\{\lambda_{\max}(E^T PE), \lambda_{\max}(H^T QH)\}$. Define

$$J_M = \sum_{n=0}^{M} [Z(nT)^T Z(nT) - \gamma^2 W(nT)^T W(nT)] \tag{17}$$

where M is a positive integer scalar. Now, noting the zero initial condition and (12) and adding the left-hand side of (13) into the right-hand side of the inequality (12), one has

$$\begin{aligned} J_M &= \sum_{n=0}^{M} [Z(nT)^T Z(nT) - \gamma^2 W(nT)^T W(nT) + V((n+1)T) - V(nT)] \\ &\quad - V((N+1)T) \\ &\le \sum_{n=0}^{\infty} \xi(nT)^T \Pi \xi(nT) \end{aligned} \tag{18}$$

where $\xi(nT) = [\hat{\xi}(nT)^T, W(nT)^T]^T$ and

$$\Pi = \begin{bmatrix} \Pi_{11} + A^T PA + L_1^T L_1 & L_1^T L_2 + A^T PB + T_1^T N_2^T & L_1^T L_3 + A^T PC + N_1 T_2 + T_1^T N_3^T \\ * & L_2^T L_2 + B^T PB - Q & L_2^T L_3 + B^T PC + N_2 T_2 \\ * & * & L_3^T L_3 + C^T PC + sym\{N_3 T_2\} - \gamma^2 I \end{bmatrix} \tag{19}$$

Now, by the Schur complement formula, it follows from (5) that $\Pi < 0$, which together with (18) ensures that (4) holds under the zero initial condition. Moreover, the condition $\Pi < 0$ implies $\hat{\Pi} < 0$. Therefore, from (14) and (16) it is easily concluded that the system (3) is asymptotically stable. ■

3.2 Control design

This subsection is devoted to the design of the local capacity H_∞ control gain k_c by using the results in Theorem 1. Obviously, the matrix inequality (5) includes multiplication of the matrices P, N_i and the control gain k_c. In the literature, more attention has been paid to the problems having this nature [41]. In the sequel, it is shown that, based on the Finsler's Lemma a convex programming algorithm in terms of LMIs is developed to solve the bilinear matrix inequality (5).

Theorem 2. For prescribed $\gamma > 0$ with $d_1, d_2 > 0$, there exist a local capacity H_∞ controller in the form of (1g) such that the system (3) is asymptotically stable and with an H_∞ performance γ for any delay satisfying (2), if there exist matrices N_1, N_2, N_3 and positive-definite matrices P and Q such that the following LMI is feasible,

$$\vartheta^{\perp T} \hat{\Pi}\, \vartheta^{\perp} < 0 \tag{20}$$

where

$$\hat{\Pi} := \begin{bmatrix} \hat{\Pi}_{11} & \hat{T}_1^T N_2^T & N_1\hat{T}_2 + \hat{T}_1^T N_3^T & A_1^T P & L_1^T \\ * & -Q & N_2\hat{T}_2 & B^T P & L_2^T \\ * & * & sym\{N_3\hat{T}_2\} - \gamma^2 I & C_1^T P & L_3^T \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix}$$

with $\hat{\Pi}_{11} = -E^T PE + (d_{12}+1)H^T QH + sym\{N_1\hat{T}_1\}$, $\tilde{T}_1 = [I,-I,0]$, $\hat{T}_1 = [0,0,-I]$ and $\tilde{T}_2 := [0,-I,0,I,0]$. In this case, a desired control gain k_c can be obtained from the following inequality

$$\hat{\Pi} + k_c\ sym\{\xi\,\vartheta\} < 0 \tag{21}$$

with

$$\xi := \begin{bmatrix} N_1 & 0 \\ N_2 & 0 \\ N_3 & 0 \\ 0 & P \\ 0 & 0 \end{bmatrix}, \quad \vartheta := \begin{bmatrix} \tilde{T}_1^T & A_1^T \\ 0 & 0 \\ \tilde{T}_2^T & C_1^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T, \quad \vartheta^{\perp} := \begin{bmatrix} S_1 & S_2 & 0 \\ I & 0 & 0 \\ S_3 & S_4 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix},$$

such that

$$\begin{bmatrix} \tilde{T}_1 & \tilde{T}_2 \\ A_1 & C_1 \end{bmatrix}^{\perp} = \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4 \end{bmatrix}.$$

Proof. Let $T_1 := k_c\tilde{T}_1 + \hat{T}_1$ and $T_2 := k_c\tilde{T}_2$. Then, the matrix inequality (5) can be rewritten as the inequality (21). Based on the Finsler's Lemma [42], it follows that (21) has a solution if the LMI (20) holds. ■

Table 1
Controller gain k_c w.r.t. d_2 and γ.

	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
$d_2 = 1$	0.6290	0.6705	0.7110
$d_2 = 2$	0.4185	0.4550	0.4955
$d_2 = 3$	0.2010	0.2725	0.3015

IV. NUMERICAL RESULTS

Consider the case of a supplier of components to the automotive industry and for which production data documents orders flowing between five work systems over a 162-day period. In this network, all order flows are unidirectional; therefore, the fundamental dynamic properties of capacity adjustment in the individual work systems are independent. Then, the internal flow of orders is approximated using the following matrix [6],

$$R = \begin{bmatrix} 0 & 106/341 & 235/341 & 0 & 0 \\ 0 & 0 & 0 & 188/401 & 204/401 \\ 0 & 0 & 0 & 100/236 & 129/236 \\ 0 & 0 & 0 & 0 & 268/295 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

in which element R_{ij} is the total number of orders that went from work system i to work system j divided by the total number of orders that left work system i.

Consider $L_1 = [I \quad -R^T \quad 0]$ and $L_2 = L_3 = 0$ in (1) with the sampling time $T = 1$ scd. It is required to find a controller gain k_c in (1e) such that the system (3a) is asymptotically stable and the H_∞ performance measure is satisfied as well. To this end, in light of Theorems 1 and 2, the LMIs (20)-(21) using Matlab LMI Control Toolbox for different values of parameter d_2 with $d_1 = 0$, and different values of the H_∞ performance bound γ, are solved and the values of the parameter k_c are obtained and shown in Table 1.

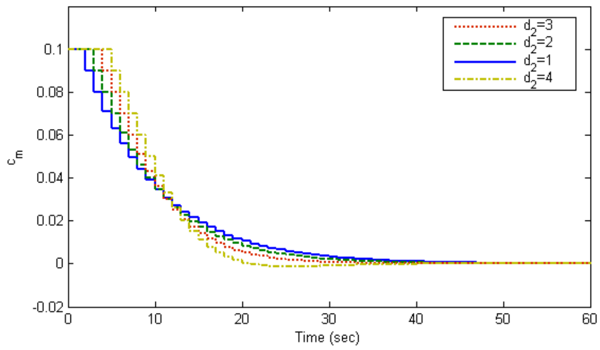

Fig. 2. Time behaviour of the local capacity changes at the shearing-sawing work system.

For simulation purposes, changes in the local capacity ($c_m(nT)$) of the work systems are considered under the controller gain $k_c = 0.1\,scd^{-1}$ and the H_∞ performance bound $\gamma = 0.15$. In this case, in response to a 1-order step planned levels ($wip_p(nT)$) at the shearing-sawing work, time behaviour of the local capacity changes at the shearing-sawing work system is depicted in Figure 2 for four different values of the upper bound of the time-varying delay $d(n)$, i.e. $d_2 = \{1,2,3,4\}$.

V. CONCLUSION

The problem of local capacity H_∞ control for a class of production networks of autonomous work systems with time-varying delays in the capacity changes was investigated in this paper. The system under consideration was modelled as a discrete-time singular form. Attention was focused on the design of a controller gain for the local capacity adjustments which maintains the work in progress in each work system in the vicinity of planned levels and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. In terms of a matrix inequality, a sufficient condition for the solvability of this problem was presented using an appropriate Lyapunov function, which is dependent on the size of the delay and is solved by existing convex optimization techniques. When this matrix inequality is feasible, the controller gain can be found by using LMI Toolbox Matlab.

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