A convex optimisation approach to robust observer-based H_∞ control design of linear parameter-varying delayed systems

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Abstract: This paper presents a convex optimisation method for observer-based control design of LPV neutral systems. Utilising the polynomials parameter-dependent quadratic functions and a suitable change of variables, the required sufficient conditions with high precision for the design of a desired observer-based control are established in terms of delay-dependent parameter-independent linear matrix inequalities. An observer-based controller guaranteeing asymptotic stability of the closed-loop system and satisfying a prescribed level of performance to the LPV neutral system with constant delay parameters is developed. A Lyapunov-Krasovskii method underlies the observer-based H_∞ control design. A numerical example with simulation results illustrates the effectiveness of the methodology.

Keywords: LPV systems; observer-based control; time-delay; linear matrix inequality; LMI.

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1 Introduction

The problem of observer design for reconstructing state variables is a more involved issue in systems with any kind of delay. In general, some sufficient conditions for the existence of an observer have been established and computational algorithms for construction of the observers have been presented in the literature (see Busawon and Saif, 1999; Fridman et al., 2003; Hassibi et al., 1999; De Souza et al., 2000; Thau, 1973; Trinh et al., 2006). Lyapunov stability theory is used to design the state observers for linear time-varying or nonlinear systems (see Busawon and Saif, 1999; Trinh et al., 2006; Gao and Wang, 2003, 2004; Geromel and de Oliveira, 2001; Gu and Poon, 2001). Recently, problem of guaranteed-cost observer-based control was studied in Lien (2005) for a class of uncertain neutral time-delay systems such that the convex optimisation problem is formulated in terms of linear matrix inequalities (LMIs) and an equality constraint which are not in the classis LMI solvable form.

On the other hand, the stability analysis, filtering and control design of linear parameter-varying (LPV) systems where the state-space matrices depend on parameter vector, whose values are not known a priori, but can be measured online, have received considerable attention recently (Apkarian and Gahinet, 1995; Feron et al., 1996; Gahinet et al., 1996; Shamma and Athans, 1991; Zhang et al., 2002). Concerning unknown parameter vector, an adaptive method has been presented for robust stabilisation with H_∞ performance of LPV systems in Karimi (2007). It is well known that stability analysis of the LPV systems via the use of classical quadratic Lyapunov function leads to conservative results. Therefore, to investigate the stability of LPV systems one needs to resort the use of parameter-dependent Lyapunov functions to achieve necessary and sufficient conditions. Generally, the robust stability analysis of LPV systems is an NP-hard problem because parameter-dependent LMIs (PLMIs) (Blondel and Tsitsiklis, 1997). Recently, Bliman (2004a) states that PLMIs admit polynomial solutions; provided they are feasible for each parameter value. This result paved the way to non-conservative conditions for LPV and LTI parameter-dependent (LTIPD) systems (Chesi et al., 2005; Karimi, 2006a). Therefore, a systematic way for the use of polynomial parameter-dependent quadratic (PPDQ) functions in the state/output feedback control of LTI parameter-dependent systems with time-delay in the state vector was proposed in Karimi (2006b) and Karimi et al. (2005). It is noted that the above paper introduces a delay-independent stability criterion which is a source of conservativeness in comparison with the present paper. The stability and the performance issues of the LPV systems with delay are then both theoretically and practically important and are a field of intense research. In general, the presence of a delay in a system may be the result of some essential simplification of the corresponding process model. Therefore, the delay effects problem on the stability of systems including delays in the state and/or the input is a problem of recurring interest since the delay presence may induce complex behaviours (oscillation, instability, bad performances) for the system (see Karimi, 2006c; Niculesu, 2001). Recently, some appreciable researches have been performed to analyse and to synthesise LPV time-delay systems (see e.g., Zhang et al., 2002; Tan et al., 2003; Wu and Grigoriadis, 2001). It is also worth citing that few studies have been done for the design of robust H_∞ filters for LPV systems (Mahmoud and Boujarwah, 2001; Velni and Grigoriadis, 2007; Wu et al., 2006). Up to now, to the best of our knowledge, no results about the observer-based H_∞ control problem of LPV neutral systems which are in the classis LMI solvable form are available in the literature and remains to be important and challenging. This motivates the present study.

In this paper, we are concerned to develop an efficient convex optimisation approach for delay-dependent observer-based H_∞ state feedback control problem of LPV neutral systems. It is assumed that the state-space data depend on parameters that are measurable in real time. Our motivation for considering such a delay-dependent parameter-dependent observer-based H_∞ control design is that in some cases utilising the available information for the measured parameters and delay parameters is a natural way to reduce the conservatism of the design. The main merit of the proposed method is the fact that it provides a convex problem by a suitable change of variables such the observer and the control gains can be found from a PLMI formulation. Then, new required sufficient conditions are established in terms of delay-dependent LMIs combined with the Lyapunov-Krasovskii method for the existence of the desired delay-dependent observer-based H_∞ control such that the resulting observer error system is asymptotically stable and satisfies a prescribed level of H_∞ performance measure. It is mentionable that application of the PPDQ functions and using the parameter-dependent Kalman-Yakubovich-Popov (KYP) lemma (see Gusev, 2006) make it possible to relax the PLMIs into conventional (parameter-independent) LMIs by introducing some Lagrange multiplier matrices. Eventually, an illustrative example is given to show the qualification of our filter design methodology.

1.1 Notation

The notations used throughout the paper are fairly standard. The matrices I_n, 0_n, $0_{n\times p}$ are the identity matrix and the $n \times n$ and $n \times p$ zero matrices, respectively. $\overline{\sigma}(A)$ denotes to the largest singular value of the matrix A and the operator $sym(A)$ represents $A+A^T$. The symbol * denotes the elements below the main diagonal of a symmetric block matrix. Also, the symbol ⊗ denotes Kronecker product, the power of Kronecker products being used with the natural meaning

$$M^{0\otimes} = 1,\ M^{p\otimes} := M^{(p-1)\otimes} \otimes M$$

and

$$\otimes_{i=m}^{1} \rho_i^{[k]} := \rho_m^{[k]} \otimes \cdots \otimes \rho_1^{[k]}.$$

Let

$$\hat{J}_k, \breve{J}_k, \breve{J}_{n,k}^{(m,i)}, \bar{J}_{n,k}^{(m,i)} \text{ and } \vartheta^{[k]}$$

be defined by

$$\hat{J}_k := \begin{bmatrix} I_k & 0_{k\times 1} \end{bmatrix},\ \breve{J}_k := \begin{bmatrix} 0_{k\times 1} & I_k \end{bmatrix},$$

$$\breve{J}_{n,k}^{(m,i)} := \hat{J}_k^{(m-i+1)\otimes} \otimes I_{(k+1)^{i-1}n},$$

$$\bar{J}_{n,k}^{(m,i)} := \hat{J}_k^{(m-i)\otimes} \otimes \breve{J}_k \otimes I_{(k+1)^{i-1}n}$$

and

$$\vartheta^{[k]} := col\{1, \vartheta, \cdots, \vartheta^{k-1}\}, \text{ respectively,}$$

which have essential roles for polynomial manipulations. The parameter-dependent matrix $A(\rho)$ is represented shortly as $A_\rho := A(\rho)$. Finally, given a signal $x(t)$, $\|x(t)\|_2$ denotes the L_2 norm of $x(t)$; i.e., $\|x(t)\|_2^2 = \int_0^\infty x(t)^T x(t)\,dt$ and the operator T_{yu} is the transfer function mapping input $u(t)$ to output $y(t)$.

2 Problem description

We consider a class of LPV neutral systems represented by

$$\dot{x}(t) = A(\rho)\,x(t) + A_1(\rho)\,x(t-h) + A_2(\rho)\,\dot{x}(t-\tau) + B(\rho)\,u(t) + E_1(\rho)\,w(t) \tag{1a}$$

$$x(t) = \phi(t) \qquad t \in [-\max\{h,\tau\},\, 0] \tag{1b}$$

$$z(t) = L(\rho)\,x(t) \tag{1c}$$

$$y(t) = C(\rho)\,x(t) + C_1(\rho)\,x(t-h) + E_2(\rho)\,w(t) \tag{1d}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^l$, $w(t) \in L_2^s[0,\infty)$, $z(t) \in \Re^z$ and $y(t) \in \Re^p$ are state vector, control signal, disturbance input, estimated output and measured output, respectively. The time-varying function $\phi(t)$ is continuous vector valued initial function and the time delays h and τ are constant and known. The dynamical system (1) arises naturally in a wide range of applications, including: networked control systems, control of large flexible space structures, control of mechanical multi-body systems, robotics control, vibration control in structural dynamics, linear stability of flows in fluid mechanics and electrical circuit simulation (see for instance Bolea et al., 2006; Balas, 1982; Bhaya and Desoer, 1985).

Throughout the paper, we make the following assumptions.

Assumption 1: The vector-valued parameter ρ evolves continuously over time, and its range is limited to a compact subset $\zeta \subset \Re^m$ and its time derivative is bounded and satisfies $-\upsilon_i \le \dot{\rho}_i(t) \le \upsilon_i$ for $i = 1, 2, \cdots, m$.

Assumption 2: The columns of the matrix $B(\rho)$ are linearly independent.

In (1), the parameter-dependent coefficient matrices are real continuous matrix functions which affinely depend on the vector-valued parameter ρ, that are

$$\begin{bmatrix} A(\rho) & A_1(\rho) & A_2(\rho) & B(\rho) & E_1(\rho) \\ L(\rho) & 0 & 0 & 0 & 0 \\ C(\rho) & C_1(\rho) & 0 & 0 & E_2(\rho) \end{bmatrix} = \begin{bmatrix} A_0 & A_{10} & A_{20} & B_0 & E_{10} \\ L_0 & 0 & 0 & 0 & 0 \\ C_0 & C_{10} & 0 & 0 & E_{20} \end{bmatrix} + \sum_{j=1}^m \rho_j \begin{bmatrix} A_j & A_{1j} & A_{2j} & B_j & E_{1j} \\ L_j & 0 & 0 & 0 & 0 \\ C_j & C_{1j} & 0 & 0 & E_{2j} \end{bmatrix}, \tag{2}$$

In this paper, the author's attention will be focused on the design of a full order delay-dependent parameter-dependent observer-based H_∞ control with the following state-space equations

$$\dot{\hat{x}}(t) = F(\rho)\,\hat{x}(t) + F_1(\rho)\,\hat{x}(t-h) + F_2(\rho)\,\dot{\hat{x}}(t-\tau) + G(\rho)\,y(t) \tag{3a}$$

$$\hat{x}(t) = 0 \qquad t \in [-\max\{h,\tau\},\, 0] \tag{3b}$$

$$\hat{z}(t) = K_1(\rho)\,\hat{x}(t) \tag{3c}$$

$$u(t) = K(\rho)\,\hat{x}(t) \tag{3d}$$

where the parameter-dependent matrices $F(\rho), F_1(\rho), F_2(\rho), G(\rho), K(\rho)$ and $K_1(\rho)$ of the appropriate dimensions are the objectives of the control design to be determined. In (3), it is assumed that $\hat{x}(t) \in \Re^n$ is the estimation of the plant's state. The augmented system formed by (1) and (3), namely observer error system, with the estimation error $e(t) = z(t) - \hat{z}(t)$ is written in the compact form:

$$\dot{X}(t) = \bar{A}\,X(t) + \bar{A}_1 X(t-h) + \bar{A}_2\,\dot{X}(t-\tau) + \bar{E}_1\,w(t) \tag{4a}$$

$$X(t) = [\phi^T(t), 0]^T \qquad t \in [-\max\{h,\tau\},\, 0] \tag{4b}$$

$$e(t) = \bar{L}\,X(t) \tag{4c}$$

where

$$X(t) = col\{x(t), \hat{x}(t)\},\ \bar{A} = \begin{bmatrix} A_\rho & B_\rho K_\rho \\ G_\rho C_\rho & F_\rho \end{bmatrix},$$

$$\bar{A}_1 = \begin{bmatrix} A_{1\rho} & 0 \\ G_\rho C_{1\rho} & F_{1\rho} \end{bmatrix},\ \bar{A}_2 = \begin{bmatrix} A_{2\rho} & 0 \\ 0 & F_{2\rho} \end{bmatrix},$$

$$\bar{E}_1 = \begin{bmatrix} E_{1\rho} \\ G_\rho E_{2\rho} \end{bmatrix}, \text{ and } \bar{L} = [L_\rho \quad -K_{1\rho}].$$

The H_∞ norm of the system (4), by assuming the frozen LPV parameters, is given by

$$\|T_{ew}\|_\infty = \sup_{\omega \in \Re} \bar{\sigma}\{H(j\omega)\} \tag{5}$$

where

$$H(j\omega) = \bar{L}(j\omega I_n - \bar{A} - \bar{A}_1 e^{-j\omega h} - j\omega \bar{A}_2 e^{-j\omega\tau})^{-1} \bar{E}_1.$$

Definition 1: The delay-dependent observer-based H_∞ control of the type (3) is said to guarantee robust disturbance attenuation if under zero initial condition

$$\sup_{\rho \in \zeta}\ \sup_{\|w\|_2 \neq 0} \|T_{ew}\|_\infty \le \gamma \tag{6}$$

for all bounded energy disturbances and a prescribed positive value γ.

Clearly, since $\|H(j\omega)\|_\infty = \|H^T(-j\omega)\|_\infty$ for all transfer function matrices $H(s)$ with real coefficients, it follows that the H_∞ norm of the system (4) is equal to the H_∞ norm of the following system

$$-\dot{\overline{X}}(s) = \overline{A}^T\,\overline{X}(s) + \overline{A}_1^T\,\overline{X}(s+h) + \overline{A}_2^T\,\dot{\overline{X}}(s+\tau) + \overline{L}^T\,\overline{e}(s) \tag{7a}$$

$$\widetilde{w}(s) = \overline{E}_1^T\,\overline{X}(s) \tag{7b}$$

Note that the latter system represents the backward adjoint of the system (4) (Fridman et al., 2003). Its forward representation (7) is described by

$$\dot{\tilde{X}}(t) = \overline{A}^T\,\tilde{X}(t) + \overline{A}_1^T\,\tilde{X}(t-h) + \overline{A}_2^T\,\dot{\tilde{X}}(t-\tau) + \overline{L}^T\,\tilde{e}(t) \tag{8a}$$

$$\widetilde{w}(t) = \overline{E}_1^T\,\widetilde{X}(t) \tag{8b}$$

Since the characteristic equation of the systems (7) and (4) are identical, the former system is asymptotically stable if and only if the system (4) is as well.

Now, using the Newton-Leibniz formula, i.e.

$$\widetilde{X}(t) - \widetilde{X}(t-h) = \int_{t-h}^{t} \dot{\widetilde{X}}(s)\ ds,$$

and following the references Fridman and Shaked (2002) and Han (2004), the authors represent (8) in the following descriptor form

$$E\begin{bmatrix}\dot{\tilde{X}}(t)\\ \dot{\eta}(t)\end{bmatrix} = \tilde{A}\begin{bmatrix}\tilde{X}(t)\\ \eta(t)\end{bmatrix} - \begin{bmatrix}0\\ \overline{A}_1^T\end{bmatrix}\int_{t-h}^{t}\eta(s)\,ds + \begin{bmatrix}0\\ \overline{A}_2^T\end{bmatrix}\eta(t-\tau) + \begin{bmatrix}0\\ \overline{L}^T\end{bmatrix}\tilde{e}(t) \tag{9a}$$

$$\widetilde{w}(t) = \overline{E}_1^T\,\widetilde{X}(t) \tag{9b}$$

where $E = \begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix}$ and $\widetilde{A} = \begin{bmatrix} 0 & I\\ \overline{A}^T + \overline{A}_1^T & -I\end{bmatrix}$.

The problem to be addressed in this paper is formulated as follows: Given a prescribed level of disturbance attenuation γ>0, find an observer-based H_∞ control of the form (3) such that

1. the descriptor system (9) is asymptotically stable for any time delays h and τ and for all admissible parameters $\rho \in \zeta$
2. under zero initial conditions and for all non-zero $\widetilde{e}(t) \in L_2[0,\infty)$ and $\rho \in \zeta$, the induced L_2–norm of the operator form $\widetilde{e}(t)$ to the controlled output $\widetilde{w}(t)$ is less than γ.

In this case, the LPV system (1) with the observer-based control (3) is said to be robustly asymptotically stable with H_∞ performance measure.

Note that now one important role to investigating the Lyapunov-based stability of the augmented system (9) will be played by the search for polynomially parameter-dependent quadratic functions chosen within the following class.

Definition 2 (Bliman, 2004b): A polynomially parameter-dependent quadratic (PPDQ) function is said to be any quadratic function $x^T(t)\,S(\rho)\,x(t)$ on $\Re^n$ such that S(ρ) is defined as

$$S(\rho) := (\otimes_{i=m}^{1}\rho_i^{[k]} \otimes I_n)^T\, S_k\,(\otimes_{i=m}^{1}\rho_i^{[k]} \otimes I_n) \tag{10}$$

for a certain $S_k \in \Re^{k^m n \times k^m n}$ which can be divided into $k^m \times k^m$ blocks, each of dimension $n \times n$. The integer $k-1$ is called the degree of the PPDQ function $S(\rho)$.

Notice that the expression $\otimes_{i=m}^{1}\rho_i^{[k]}$ gathers in a column all the monomials with degree at most $k-1$ in each of the components of ρ.

Remark 1: According to the results of Zhang (2005), the matrix $S(\rho)$ in (10) can be written as follows:

$$S(\rho) := \sum_{|\hat{k}|=0}^{\bar{k}} \rho_1^{k_1}\cdots\rho_m^{k_m}\, S_{k_1,\cdots,k_m}$$

where $|\hat{k}| := \sum_{i=1}^{m} k_i$ with $k_i \in \mathrm{N}\cup\{0\}$ and $k = \left[\frac{\bar{k}}{2}\right] + 1$.

Notice that the nonzero blocks of the matrix S_k are just the matrix coefficients of the polynomial matrix $S(\rho)$.

3 Robust observer-based control design

In this section, we will concentrate ourselves on the determination of the observer-based control gains using Lyapunov functional method. In the literature, extensions of the quadratic Lyapunov functions to the Lyapunov-Krasovskii functionals have been proposed for time-delayed systems (see for instance Niculesu, 2001; Krasovskii, 1963). Hence, a class of Lyapunov-Krasovskii functionals for this purpose is given by

$$V(t) = \begin{bmatrix}\tilde{X}(t)\\ \eta(t)\end{bmatrix}^T EP\begin{bmatrix}\tilde{X}(t)\\ \eta(t)\end{bmatrix} + \int_{t-h}^{t}\tilde{X}^T(\sigma)Q_{1\rho}\tilde{X}(\sigma)\,d\sigma + \int_{t-\tau}^{t}\eta^T(\sigma)Q_{2\rho}\eta(\sigma)\,d\sigma + \int_{-h}^{0}\int_{t+\theta}^{t}\eta^T(\sigma)\begin{bmatrix}0\\ \overline{A}_1^T\end{bmatrix}^T R\begin{bmatrix}0\\ \overline{A}_1^T\end{bmatrix}\eta(\sigma)\,d\sigma\,d\theta \tag{11}$$

where

$$P := \begin{bmatrix} P_{1\rho} & 0\\ P_{3\rho} & P_{2\rho}\end{bmatrix} \tag{12a}$$

and

$$EP = P^T E^T = \begin{bmatrix} P_{1\rho} & 0\\ * & 0\end{bmatrix} \geq 0 \tag{12b}$$

It is noting that the first, the second and the third terms of $V(f)$ are generally used for the delay-independent stability analysis of the LPV neutral systems. In this section, the following lemma is used to give an upper bound on the integral terms in the Lyapunov-Krasovskii derivative.

Lemma 1 (Park, 1999): For any arbitrary positive definite matrix R and the matrix M the following inequality holds:

$$-2\int_{t-h}^{t} b(s)^T a(s)\,ds \le \int_{t-h}^{t} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^T \begin{bmatrix} R & RM \\ * & (RM+I)^T R^{-1}(RM+I) \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix} ds$$

Theorem 1: *Let the state-space parameter-dependent matrices of the delay-dependent observer-based H_∞ control (3) be given. Under Assumption 1, the augmented system (4) obtained from the interconnection of the plant (1) and the filter (3) achieves, simultaneously, the asymptotic stability and H_∞ performance for a given performance bound γ in the sense of Definition 1 if there exist the scalar α, the positive definite matrices P, $\{Q_{i\rho}\}_{i=1}^{2}$, R such that the following PLMI is satisfied*

$$\begin{bmatrix} \Gamma_{11} & -\alpha P^T \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix} & P^T \begin{bmatrix} 0 \\ \overline{A}_2^T \end{bmatrix} & P^T \begin{bmatrix} 0 \\ \overline{L}^T \end{bmatrix} & h(\alpha+1)P^T \\ * & -Q_{1\rho} & 0 & 0 & 0 \\ * & * & -Q_{2\rho} & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -hR \end{bmatrix} < 0 \qquad (13)$$

where

$$\Gamma_{11} = sym(P^T \tilde{A}) + sym(\alpha P^T \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix} [I \ \ 0]) + \begin{bmatrix} \sum_{j=1}^{m} \nu_j \frac{\partial P_{1\rho}}{\partial \rho_j} + Q_{1\rho} + \overline{E}_1 \overline{E}_1^T & 0 \\ * & Q_{2\rho} + h \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}^T R \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix} \end{bmatrix}.$$

Proof: Consider the same Lyapunov-Krasovskii functional (11) and a HJI function in the form of

$$J[\tilde{X}(t), \tilde{e}(t)] = \frac{dV(t)}{dt} + \tilde{w}^T(t)\tilde{w}(t) - \gamma^2 \tilde{e}^T(t)\tilde{e}(t) \qquad (14)$$

where derivative of $V(t)$ is evaluated along the trajectory of the augmented system (9). It is well known that a sufficient condition for achieving robust disturbance attenuation is that the inequality $J[\tilde{X}(t), \tilde{e}(t)] < 0$ for every $\tilde{e}(t) \in L^2$ results in a function $V(t)$, which is strictly radially unbounded. Differentiating (11) in t we obtain:

$$\begin{aligned} J[\tilde{X}(t), \tilde{e}(t)] &= \tilde{X}^T(t)\,(\sum_{j=1}^{m} \nu_j \frac{\partial P_{1\rho}}{\partial \rho_j})\tilde{X}(t) + 2\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix}^T P^T \{\tilde{A}\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{A}_2^T \end{bmatrix}\eta(t-\tau) + \begin{bmatrix} 0 \\ \overline{L}^T \end{bmatrix}\tilde{e}(t)\} + \kappa(t) \\ &+ \tilde{X}^T(t)Q_{1\rho}\tilde{X}(t) - \tilde{X}^T(t-h)Q_{1\rho}\tilde{X}(t-h) + \eta^T(t)Q_{2\rho}\eta(t) - \eta^T(t-\tau)Q_{2\rho}\eta(t-\tau) + h\eta^T(t) \\ &\times \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}^T R \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}\eta(t) - \int_{t-h}^{t} \eta^T(\sigma)\begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}^T R \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}\eta(\sigma)\, d\sigma + \tilde{X}^T(t)\overline{E}_1 \overline{E}_1^T \tilde{X}(t) - \gamma^2 \tilde{e}^T(t)\tilde{e}(t) \end{aligned} \qquad (15)$$

where

$$\kappa(t) := -2\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix}^T P^T \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix} \int_{t-h}^{t} \eta(s)\,ds.$$

Using Lemma 1 for $a(s) = \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}\eta(s)$ and $b(s) = P\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix}$ we obtain:

$$\begin{aligned} \kappa(t) \le h &\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix}^T P^T (RM + I_{2n})^T R^{-1}(RM + I_{2n})P\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix} \\ &+ 2\begin{bmatrix} \tilde{X}(t) \\ \eta(t) \end{bmatrix}^T P^T M^T R \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}(\tilde{X}(t) - \tilde{X}(t-h)) \\ &+ \int_{t-h}^{t} \eta(s)^T \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}^T R \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix}\eta(s)\,ds \end{aligned} \qquad (16)$$

From (15) and (16), it follows that

$$J[\tilde{X}(t), \tilde{e}(t)] \le \chi^T(t)\Gamma\chi(t) \qquad (17)$$

where the vector

$$\chi(t) := col\{\tilde{X}(t), \eta(t), \tilde{X}(t-h), \eta(t-\tau), \tilde{e}(t)\}$$

is an augmented state and the matrix Γ is given by

$$\Gamma = \begin{bmatrix} \hat{\Gamma}_{11} & -P^T M^T R \begin{bmatrix} 0 \\ \overline{A}_1^T \end{bmatrix} & P^T \begin{bmatrix} 0 \\ \overline{A}_2^T \end{bmatrix} & P^T \begin{bmatrix} 0 \\ \overline{L}^T \end{bmatrix} \\ * & -Q_{1\rho} & 0 & 0 \\ * & * & -Q_{2\rho} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \qquad (18)$$

where

$$\hat{\Gamma}_{11} = sym(P^T \tilde{A}) + sym\left(P^T M^T R \begin{bmatrix} 0 \\ \bar{A}_1^T \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix}\right) + hP^T (RM + I_{2n})^T R^{-1} (RM + I_{2n}) P$$

$$+ \begin{bmatrix} \sum_{j=1}^{m} \nu_j \dfrac{\partial P_{1\rho}}{\partial \rho_j} + Q_{1\rho} + \bar{E}_1 \bar{E}_1^T & 0 \\ * & Q_{2\rho} + h \begin{bmatrix} 0 \\ \bar{A}_1^T \end{bmatrix}^T R \begin{bmatrix} 0 \\ \bar{A}_1^T \end{bmatrix} \end{bmatrix}.$$

From (17), it is clear that the inequality Γ<0 is a sufficient condition to satisfy the inequality $J[\tilde{X}(t), \tilde{e}(t)] < 0$. By Schur, complements and considers $RM = \alpha I_{2n}$ for the arbitraries matrices $R = R^T > 0$ and M to remove the present nonlinearities in the matrix (18), the inequality Γ<0 results in the inequality matrix (13).

Remark 2: Theorem 1 provides a sufficient condition for the singular delay system (9) to be stable. Furthermore, by comparing Theorem 1 with Xu et al. (2002), we can regard Theorem 1 as an extension of existing results on singular systems with discrete delay to singular delay systems with both discrete and distributed delays.

Remark 3: It is easy to see that the inequality (13) implies $\Gamma_{11} < 0$. Hence by Proposition 4.2 in Fridman and Shaked (2002), the matrix P is non-singular. Then, according to the structure of the matrix P in (12), the matrix $X := P^{-1}$ has the form

$$X = \begin{bmatrix} X_{1\rho} & 0 \\ X_{3\rho} & X_{2\rho} \end{bmatrix} \tag{19}$$

where $X_{i\rho} = P_{i\rho}^{-1}$ ($i = 1, 2$), $X_{3\rho} = -X_{2\rho} P_{3\rho} X_{1\rho}$ and it can be easily seen that $\dot{X}_{1\rho} = -X_{1\rho} \dot{P}_{1\rho} X_{1\rho}$.

Now, we are in a position to give our main results on the existence of a delay-dependent observer-based control in the form of (3), and show how to construct such a desired control for the LPV neutral system in (1).

Theorem 2: *Let the positive integer $k-1$ as the degree of the PPDQ functions is given. Under Assumptions 1 and 2, consider the LPV neutral system (1) with the known constant time-delay parameters $h, \tau > 0$. For a given performance bound γ, there exits a delay-dependent observer-based H_∞ control in the form of (3) such that the resulting closed-loop system is robustly asymptotically stable and satisfies H_∞ performance measure in the sense of Definition 1, if there exist a scalar α, the set of matrices $W_\rho, W_{1\rho}, \cdots, W_{4\rho}$ and the set of positive-definite matrices $X_{11\rho}^{(1)}, X_{11\rho}^{(2)}, X_{11\rho}^{(3)}$, $X_{22\rho}^{(1)}, Q_{11\rho}^{(2)}, Q_{22\rho}^{(2)}, \bar{R}, \hat{Q}_{1\rho}$, satisfying the PLMIs*

$$\begin{bmatrix} Q_{11\rho}^{(2)} & Q_{22\rho}^{(2)} \\ * & Q_{22\rho}^{(2)} \end{bmatrix} > 0 \tag{20a}$$

$$\begin{bmatrix} \Sigma_{11} & -\alpha \begin{bmatrix} 0 \\ \begin{bmatrix} A_\rho^T X_{11\rho}^{(1)} & C_{1\rho}^T W_{3\rho} \\ 0 & W_{1\rho} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ \begin{bmatrix} A_{2\rho}^T Q_{11\rho}^{(2)} & A_{2\rho}^T Q_{22\rho}^{(2)} \\ W_{2\rho} & W_{2\rho} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ \begin{bmatrix} L_\rho^T \\ -K_{1\rho}^T \end{bmatrix} \end{bmatrix} \\ * & -\hat{Q}_{1\rho} & 0 & 0 \\ * & * & -\begin{bmatrix} Q_{11\rho}^{(2)} & Q_{22\rho}^{(2)} \\ * & Q_{22\rho}^{(2)} \end{bmatrix} & 0 \\ * & * & * & -\gamma^2 I \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix} h(\alpha+1)\overline{R}^T & \begin{bmatrix}\begin{bmatrix} X_{11\rho}^{(3)} & X_{22\rho}^{(1)} \\ X_{22\rho}^{(1)} & X_{22\rho}^{(1)} \end{bmatrix} \\ \begin{bmatrix} X_{11\rho}^{(2)} & X_{22\rho}^{(2)} \\ X_{22\rho}^{(2)} & X_{22\rho}^{(2)} \end{bmatrix}\end{bmatrix} & h\begin{bmatrix} 0 & \begin{bmatrix} X_{11\rho}^{(3)^T} A_\rho + W_{3\rho}^T C_{1\rho} & W_{1\rho}^T \\ X_{22\rho}^{(1)} A_\rho + W_{3\rho}^T C_{1\rho} & W_{1\rho}^T \end{bmatrix} \\ 0 & \begin{bmatrix} X_{11\rho}^{(2)^T} A_\rho + W_{3\rho}^T C_{1\rho} & W_{1\rho}^T \\ X_{22\rho}^{(1)} A_\rho + W_{3\rho}^T C_{1\rho} & W_{1\rho}^T \end{bmatrix}\end{bmatrix} & \begin{bmatrix}\begin{bmatrix} X_{11\rho}^{(1)} E_{1\rho} \\ W_{3\rho}^T E_{2\rho} \end{bmatrix} \\ 0 \end{bmatrix} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -h\overline{R} & 0 & 0 & 0 \\ * & -\begin{bmatrix} Q_{11\rho}^{(2)} & Q_{22\rho}^{(2)} \\ * & Q_{22\rho}^{(2)} \end{bmatrix} & 0 & 0 \\ * & * & -h\overline{R} & 0 \\ * & * & * & -I \end{bmatrix} < 0 \qquad (20b)$$

where

$$\Sigma_{11} = sym\left(\begin{bmatrix} & \begin{bmatrix} X_{11\rho}^{(3)} & X_{22\rho}^{(1)} \\ X_{22\rho}^{(1)} & X_{22\rho}^{(1)} \end{bmatrix} & \begin{bmatrix} X_{11\rho}^{(2)} & X_{22\rho}^{(1)} \\ X_{22\rho}^{(1)} & X_{22\rho}^{(1)} \end{bmatrix} \\ \begin{bmatrix} (A_\rho + A_{1\rho})^T X_{11\rho}^{(1)} - X_{11\rho}^{(3)} & (C_\rho + C_{1\rho})^T W_{3\rho} - X_{22\rho}^{(1)} \\ W_{4\rho} - X_{22\rho}^{(1)} & W_\rho^T + W_{1\rho}^T - X_{22\rho}^{(1)} \end{bmatrix} & & -\begin{bmatrix} X_{11\rho}^{(2)} & X_{22\rho}^{(1)} \\ X_{22\rho}^{(1)} & X_{22\rho}^{(1)} \end{bmatrix} \end{bmatrix}\right)$$

$$+ sym\left(\alpha\begin{bmatrix} 0 & 0 \\ \begin{bmatrix} A_{1\rho}^T X_{11\rho}^{(1)} & C_{1\rho}^T W_{3\rho} \\ 0 & W_{1\rho} \end{bmatrix} & 0 \end{bmatrix}\right) + \begin{bmatrix} \begin{bmatrix} -\sum_{j=1}^{m} \nu_j \dfrac{\partial X_{11\rho}^{(1)}}{\partial \rho_j} & 0 \\ * & -\sum_{j=1}^{m} \nu_j \dfrac{\partial X_{22\rho}^{(1)}}{\partial \rho_j} \end{bmatrix} + \hat{Q}_{1\rho} & 0 \\ * & 0 \end{bmatrix},$$

then the state-space matrices of the observer-based H_∞ *control are given by*

$$F_\rho = X_{22\rho}^{(1)^{-1}} W_\rho^T, \quad F_{1\rho} = X_{22\rho}^{(1)^{-1}} W_{1\rho}^T, \quad F_{2\rho} = Q_{22\rho}^{(2)^{-1}} W_{2\rho}^T, \qquad (21)$$
$$G_\rho = X_{22\rho}^{(1)^{-1}} W_{3\rho}^T, \quad K_\rho = (B_\rho^T B_\rho)^{-1} B_\rho^T X_{11\rho}^{(1)^{-1}} W_{4\rho}^T$$

Proof: Let

$$\zeta = diag\,\{X^T,\ X_{1\rho}, \overline{Q}_{2\rho}, I, \overline{R}\} \qquad (22)$$

where $\overline{Q}_{2\rho} = Q_{2\rho}^{-1}$ and $\overline{R} = R^{-1}$. By premultiplying ζ and postmultiplying ζ^T to the matrix inequality (13) in Theorem 1, we obtain (by Schur complements and considering $\sum_{j=1}^{m} \nu_j \frac{\partial X_{1\rho}}{\partial \rho_j} = -X_{1\rho}(\sum_{j=1}^{m} \nu_j \frac{\partial P_{1\rho}}{\partial \rho_j}) X_{1\rho}$ from Remark 3).

$$\begin{bmatrix} \hat{\Sigma}_{11} & -\alpha\begin{bmatrix}0\\ \overline{A}_1^T\end{bmatrix}X_{1\rho} & \begin{bmatrix}0\\ \overline{A}_2^T\end{bmatrix}\overline{Q}_{2\rho} & \begin{bmatrix}0\\ \overline{L}^T\end{bmatrix} & h(\alpha+1)\overline{R}^T & \begin{bmatrix}X_{3\rho}^T\\ X_{2\rho}^T\end{bmatrix} & h\begin{bmatrix}X_{3\rho}^T\\ X_{2\rho}^T\end{bmatrix}\begin{bmatrix}0\\ \overline{A}_1^T\end{bmatrix}^T & \begin{bmatrix}X_{1\rho}\overline{E}_1\\ 0\end{bmatrix} \\ * & -\hat{Q}_{1\rho} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\overline{Q}_{2\rho} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -h\overline{R} & 0 & 0 & 0 \\ * & * & * & * & * & -\overline{Q}_{2\rho} & 0 & 0 \\ * & * & * & * & * & * & -h\overline{R} & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (23)$$

where $\hat{Q}_{1\rho} := X_{1\rho}Q_{1\rho}X_{1\rho}$ and

$$\hat{\Sigma}_{11} = sym(\widetilde{A}X) + sym(\alpha\begin{bmatrix}0\\ \overline{A}_1^T\end{bmatrix}\begin{bmatrix}I & 0\end{bmatrix}X) + \begin{bmatrix} -\sum_{j=1}^{m}\nu_j\dfrac{\partial X_{1\rho}}{\partial\rho_j} + \hat{Q}_{1\rho} & 0 \\ * & 0\end{bmatrix}.$$

Since the matrix inequality (23) is not a LMI condition owing to the multiplication of the matrix variables (Goh et al., 1996), then the following change of variables are considered in our manipulations to get a convex problem

$$W_\rho = F_\rho^T X_{22\rho}^{(1)},\quad W_{1\rho} = F_{1\rho}^T X_{22\rho}^{(1)},\quad W_{2\rho} = F_{2\rho}^T Q_{22\rho}^{(2)}, \\ W_{3\rho} = G_\rho^T X_{22\rho}^{(1)},\quad W_{4\rho} = K_\rho^T B_\rho^T X_{11\rho}^{(1)} \quad (24)$$

with

$$X_{1\rho} = \begin{bmatrix} X_{11\rho}^{(1)} & 0 \\ * & X_{22\rho}^{(1)}\end{bmatrix},\; X_{2\rho} = \begin{bmatrix} X_{11\rho}^{(2)} & X_{22\rho}^{(1)} \\ X_{22\rho}^{(1)} & X_{22\rho}^{(1)}\end{bmatrix}, \\ X_{3\rho} = \begin{bmatrix} X_{11\rho}^{(3)} & X_{22\rho}^{(1)} \\ X_{22\rho}^{(1)} & X_{22\rho}^{(1)}\end{bmatrix},\; \overline{Q}_{2\rho} = \begin{bmatrix} Q_{11\rho}^{(2)} & Q_{22\rho}^{(2)} \\ Q_{22\rho}^{(2)} & Q_{22\rho}^{(2)}\end{bmatrix} \quad (25)$$

Now, the underlying observer-based H_∞ control synthesis problem is a convex problem, therefore, the analysis condition to satisfy asymptotic stability and an H_∞ performance measure results in the LMIs (20) and the observer-based H_∞ control matrices are computed from (21).

Remark 4: Notice that the PLMIs (20) correspond to infinite-dimensional convex problems due to their parametric dependence. The major difficulty in solving LPV analysis problem lies in how the LMIs (20) will be verified over the entire parameter space. In the literature, two types of methods are usually used (see Apkarian et al., 1995; Tan et al., 2003; Wu and Grigoriadis, 2001; Apkarian and Adams, 1998; Apkarian and Gahinet, 1995; Apkarian and Tuan, 2000; Lim and How, 2002); either the controller gain is first computed for a bunch parameter values (gridding of the parameter space), and then interpolated between the nodes of this grid (but the stability, and possibly performance, results are not guaranteed between the nodes); or the solution of the parameter-dependent LMIs involved is sought for with prescribed dependence with respect to the parameters, usually constant or affine (at the cost of adding conservatism). Moreover, techniques such as the *S*-procedure, the KYP lemma or multiconvexity concepts can also be used repeatedly to get a finite number of (sufficient) LMI conditions (see Gusev, 2006, and references therein). The main approach employed here is based on applying the parameter-dependent KYP lemma by introducing some Lagrange multiplier matrices to obtain less conservative results (see Bliman, 2004a; 2005; Karimi, 2006a; Bliman, 2005) and to study polynomial solutions (with respect to the parameters) of a parameter-dependent LMI.

Lemma 2: Let the degree of the PPDQ function F_ρ be $k-1$. A PPDQ function of degree k for parameter-dependent matrix $F_\rho \Pi_\rho$ is given by

$$F_\rho \Pi_\rho := (\otimes_{i=m}^{1}\rho_i^{[k+1]} \otimes I_n)^T S_k (\otimes_{i=m}^{1}\rho_i^{[k+1]} \otimes I_q) \quad (26)$$

Where $\Pi_\rho = \Pi_0 + \sum_{i=1}^{m}\rho_i\Pi_i$ and $\Pi_i \in \Re^{n\times q}$, then the parameter-independent matrix $S_k \in \Re^{((k+1)^m n)\times((k+1)^m q)}$ which depends on the parameter-independent matrix F_k linearly is defined as

$$S_k = \vec{J}_{n,k}^{(m,1)^T} F_k\left(\hat{J}_k^{m\otimes}\otimes\Pi_0 + \sum_{i=1}^{m}\hat{J}_k^{(m-i)\otimes}\otimes\widetilde{J}_k\otimes\hat{J}_k^{(i-1)\otimes}\otimes\Pi_i\right). \quad (27)$$

where $\vec{J}_{n,k}^{(m,1)} = \vec{J}_{n,k}^{(m,i)}\Big|_{i=1} = \hat{J}_k^{m\otimes}\otimes I_n$.

Proof: See Karimi et al. (2005).

Remark 5: (PLMI relaxation) It is noted that according to Lemma 2 the product of two polynomial matrices is expressed in terms of a PPDQ function for a specific degree. It is shown in Bliman (2004a) that if an LMI polynomially depends on the parameters, then it can be transformed into an LMI of a larger dimension that does not depend on these parameters.

According to Lemma 2 for the parameter-dependent matrix A_ρ, we have

$$X_{11\rho}^{(1)}\, A_\rho := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T S_{11,k}^{(1)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (28a)$$

$$X_{11\rho}^{(2)^T}\, A_\rho := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T S_{11,k}^{(2)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (28b)$$

$$X_{11\rho}^{(3)^T}\, A_\rho := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T S_{11,k}^{(3)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (28c)$$

$$X_{22\rho}^{(1)}\, A_\rho := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T S_{22,k}^{(1)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (28d)$$

where the PPDQ functions $X_{11\rho}^{(1)}, X_{22\rho}^{(1)}$ and $X_{11\rho}^{(3)}$ of the order $k-1$ satisfy the representation form of (10), where S_k stands for the parameter-independent matrices $X_{11,k}^{(1)}, X_{22,k}^{(1)}, X_{11,k}^{(3)}$, then the parameter-independent matrices $S_{11,k}^{(1)}, S_{11,k}^{(2)}, S_{11,k}^{(3)}$ and $S_{22,k}^{(1)}$ are represented, respectively, in the following forms

$$S_{11,k}^{(1)} = \breve{J}_{n,k}^{(m,1)^T} X_{11,k}^{(1)} \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i \right) \quad (29a)$$

$$S_{11,k}^{(2)} = \breve{J}_{n,k}^{(m,1)^T} X_{11,k}^{(2)^T} \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i \right) \quad (29b)$$

$$S_{11,k}^{(3)} = \breve{J}_{n,k}^{(m,1)^T} X_{11,k}^{(3)^T} \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i \right) \quad (29c)$$

$$S_{22,k}^{(1)} = \breve{J}_{n,k}^{(m,1)^T} X_{22,k}^{(1)} \left(\hat{J}_k^{m\otimes} \otimes A_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_i \right) \quad (29d)$$

Similarly, for the matrices $A_{1\rho}, A_{2\rho}$ and $E_{1\rho}$ we have

$$X_{11\rho}^{(1)} A_{1\rho} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \hat{S}_{11,k}^{(1)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (30a)$$

$$Q_{11\rho}^{(2)} A_{2\rho} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \bar{S}_{11,k}^{(2)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (30b)$$

$$Q_{22\rho}^{(2)} A_{2\rho} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \bar{S}_{22,k}^{(2)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (30c)$$

$$X_{11\rho}^{(1)} E_{1\rho} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \tilde{S}_{11,k}^{(1)} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_s) \quad (30d)$$

where

$$Q_{11\rho}^{(2)} := (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T Q_{11,k}^{(2)} (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n),$$

$$Q_{22\rho}^{(2)} := (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)^T Q_{22,k}^{(2)} (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n),$$

and the matrices

$$\hat{S}_{11,k}^{(1)}, \bar{S}_{11,k}^{(2)}, \bar{S}_{22,k}^{(2)}$$

and

$$\tilde{S}_{11,k}^{(1)}$$

are, respectively,

$$\hat{S}_{11,k}^{(1)} = \breve{J}_{n,k}^{(m,1)^T} X_{11,k}^{(1)} \left(\hat{J}_k^{m\otimes} \otimes A_{10} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_{1i} \right) \quad (31a)$$

$$\bar{S}_{11,k}^{(2)} = \breve{J}_{n,k}^{(m,1)^T} Q_{11,k}^{(2)} \left(\hat{J}_k^{m\otimes} \otimes A_{20} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_{2i} \right) \quad (31b)$$

$$\bar{S}_{22,k}^{(2)} = \breve{J}_{n,k}^{(m,1)^T} Q_{22,k}^{(2)} \left(\hat{J}_k^{m\otimes} \otimes A_{20} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes A_{2i} \right) \quad (31c)$$

$$\tilde{S}_{11,k}^{(1)} = \breve{J}_{n,k}^{(m,1)^T} X_{11,k}^{(1)} \left(\hat{J}_k^{m\otimes} \otimes E_{10} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes E_{1i} \right) \quad (31d)$$

The parameter-dependent matrices

$$\{W_\rho, W_{1\rho}, W_{2\rho}, W_{4\rho}\} \in \Re^{n \times n} \text{ and } W_{3\rho} \in \Re^{p \times n}$$

can be also represented in terms of the PPDQ functions of the order $k-1$ in (10) where S_k stands for the parameter-independent matrices

$$\{W_k, W_{1,k}, W_{2,k}, W_{4,k}\} \in \Re^{(k^m n) \times (k^m n)}$$

and

$$W_{3,k} \in \Re^{(k^m p) \times (k^m n)}.$$

Therefore, the following representation forms are concluded

$$W_{3\rho}^T C_\rho := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \overline{W}_{3,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (32a)$$

$$W_{3\rho}^T C_{1\rho} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \widetilde{W}_{3,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (32b)$$

$$W_{3\rho}^T E_{2\rho} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T \hat{W}_{3,k} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_s) \quad (32c)$$

where the parameter-independent matrices

$$\{\overline{W}_{3,k}, \widetilde{W}_{3,k}\} \in \Re^{((k+1)^m n) \times ((k+1)^m n)}$$

and

$$\hat{W}_{3,k} \in \Re^{((k+1)^m n) \times ((k+1)^m s)}$$

are computed by

$$\overline{W}_{3,k} = \breve{J}_{n,k}^{(m,1)^T} W_{3,k}^T \left(\hat{J}_k^{m\otimes} \otimes C_0 + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes C_i \right) \quad (33a)$$

$$\widetilde{W}_{3,k} = \breve{J}_{n,k}^{(m,1)^T} W_{3,k}^T \left(\hat{J}_k^{m\otimes} \otimes C_{10} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes C_{1i} \right) \quad (33b)$$

$$\hat{W}_{3,k} = \breve{J}_{n,k}^{(m,1)^T} W_{3,k}^T \left(\hat{J}_k^{m\otimes} \otimes E_{20} + \sum_{i=1}^{m} \hat{J}_k^{(m-i)\otimes} \otimes \tilde{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes E_{2i} \right) \quad (33c)$$

Lemma 3: Let $Z(\rho) := Z_k (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n)$. Then,

$$\frac{\partial Z(\rho)}{\partial \rho_i} := Z_k (\hat{J}_k^{(m-i)\otimes} \otimes L_k \hat{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes I_n)(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n) \quad (34)$$

where

$$\Delta_k := \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 2 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & k-1 & 0 \end{bmatrix}.$$

Proof: See Bliman (2005).

Remark 6: For the parameter-dependent matrices $X_{11\rho}^{(1)}$ and $X_{22\rho}^{(1)}$ with the PPDQ representations of degree $k-1$, we obtain (using Lemma 3)

$$\frac{\partial X_{11\rho}^{(1)}}{\partial \rho_i} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T X_{11,k}^{\prime(1)_i} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)$$

with

$$X_{11,k}^{\prime(1)_i} = sym\{\overleftrightarrow{J}_{n,k}^{(m,1)^T} X_{11,k}^{(1)} (\hat{J}_k^{(m-i)\otimes} \otimes \Delta_k \hat{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes I_n)\} \quad (35)$$

and

$$\frac{\partial X_{22\rho}^{(1)}}{\partial \rho_i} := (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)^T X_{22,k}^{\prime(1)_i} (\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n)$$

with

$$X_{22,k}^{\prime(1)_i} = sym\{\overleftrightarrow{J}_{n,k}^{(m,1)^T} X_{22,k}^{(1)} (\hat{J}_k^{(m-i)\otimes} \otimes \Delta_k \hat{J}_k \otimes \hat{J}_k^{(i-1)\otimes} \otimes I_n)\} \quad (36)$$

Theorem 3: *Let the positive integer* $k-1$ *as the degree of the PPDQ functions be given. Under Assumptions 1 and 2, consider the LPV neutral system (1) with the known constant time-delay parameters* $h, \tau > 0$. *For a given performance bound* γ, *if there exist the scalar* α, *the set of positive definite matrices*

$$X_{11,k}^{(1)}, X_{22,k}^{(1)}, Q_{11,k}^{(2)}, Q_{22,k}^{(2)}, \hat{Q}_{11,k}^{(1)}, \hat{Q}_{22,k}^{(1)}, \overline{R}_1, \overline{R}_2,$$

the set of matrices

$$W_k, W_{1,k}, W_{2,k}, W_{3,k}, W_{4,k},$$

$$K_{1,k}, X_{11,k}^{(2)}, X_{11,k}^{(3)}, \hat{Q}_{12,k}^{(1)}, \overline{R}_3$$

and the set of positive definite Lagrange multiplier matrices

$$\{Z_{i,k}^{(j)}\}_{i=1,\cdots,m}^{j=1,\cdots,16} \text{ and } \{T_{i,k}^{(1)}, T_{i,k}^{(2)}\}_{i=1,\cdots,m}$$

to the following LMIs

$$\begin{bmatrix} \Psi_{11} & \overleftrightarrow{J}_{n,k}^{(m,1)^T} Q_{22,k}^{(2)} \overleftrightarrow{J}_{n,k}^{(m,1)} \\ * & \Psi_{22} \end{bmatrix} > 0 \quad (37a)$$

$$\left[\begin{array}{cccc}
\begin{bmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} & \tilde{\Sigma}_{13} & \tilde{\Sigma}_{14} \\ * & \tilde{\Sigma}_{22} & \tilde{\Sigma}_{23} & \tilde{\Sigma}_{24} \\ * & * & \tilde{\Sigma}_{33} & \tilde{\Sigma}_{34} \\ * & * & * & \tilde{\Sigma}_{44} \end{bmatrix} - \alpha \begin{bmatrix} 0 \\ \begin{bmatrix} S_{11,k}^{(1)^T} & \tilde{W}_{3,k}^T \\ 0 & \overleftrightarrow{J}_{n,k}^{(m,1)^T} W_{1,k} \overleftrightarrow{J}_{n,k}^{(m,1)} \end{bmatrix} \end{bmatrix} & & \begin{bmatrix} 0 \\ \begin{bmatrix} S_{11,k}^{(2)^T} & \overline{S}_{22,k}^{(2)^T} \\ \overleftrightarrow{J}_{n,k}^{(m,1)^T} W_{2,k} \overleftrightarrow{J}_{n,k}^{(m,1)} & \overleftrightarrow{J}_{n,k}^{(m,1)^T} W_{2,k} \overleftrightarrow{J}_{n,k}^{(m,1)} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} 0 \\ \begin{bmatrix} L_k^T \\ -\overleftrightarrow{J}_{n,k}^{(m,1)^T} K_{1,k}^T \overleftrightarrow{J}_{z,k}^{(m,1)} \end{bmatrix} \end{bmatrix} \\
* & \begin{bmatrix} \tilde{\Sigma}_{55} & -\overleftrightarrow{J}_{n,k}^{(m,1)^T} \hat{Q}_{12,k}^{(1)} \overleftrightarrow{J}_{n,k}^{(m,1)} \\ * & \tilde{\Sigma}_{66} \end{bmatrix} & 0 & 0 \\
* & * & \begin{bmatrix} \tilde{\Sigma}_{77} & -\overleftrightarrow{J}_{n,k}^{(m,1)^T} Q_{22,k}^{(2)} \overleftrightarrow{J}_{n,k}^{(m,1)} \\ * & \tilde{\Sigma}_{88} \end{bmatrix} & 0 \\
* & * & * & \tilde{\Sigma}_{99} \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{array}\right.$$

$$
\left.\begin{array}{cccc}
h(\alpha+1)\begin{bmatrix} \bar{R}_{1,k} & \bar{R}_{3,k}^{T} \\ * & \bar{R}_{2,k} \end{bmatrix} &
\begin{bmatrix} \begin{bmatrix} \vec{J}_{n,k}^{(m,1)T} X_{11,k}^{(3)} \vec{J}_{n,k}^{(m,1)} & \vec{J}_{n,k}^{(m,1)T} X_{22,k}^{(1)} \vec{J}_{n,k}^{(m,1)} \\ \vec{J}_{n,k}^{(m,1)T} X_{22,k}^{(1)} \vec{J}_{n,k}^{(m,1)} & \vec{J}_{n,k}^{(m,1)T} X_{22,k}^{(1)} \vec{J}_{n,k}^{(m,1)} \end{bmatrix} \\ \begin{bmatrix} \vec{J}_{n,k}^{(m,1)T} X_{11,k}^{(2)} \vec{J}_{n,k}^{(m,1)} & \vec{J}_{n,k}^{(m,1)T} X_{22,k}^{(2)} \vec{J}_{n,k}^{(m,1)} \\ \vec{J}_{n,k}^{(m,1)T} X_{22,k}^{(2)} \vec{J}_{n,k}^{(m,1)} & \vec{J}_{n,k}^{(m,1)T} X_{22,k}^{(2)} \vec{J}_{n,k}^{(m,1)} \end{bmatrix} \end{bmatrix} &
h\begin{bmatrix} 0 & \begin{bmatrix} S_{11,k}^{(3)} + \tilde{W}_{3,k} & \vec{J}_{n,k}^{(m,1)T} W_{1,k}^{T} \vec{J}_{n,k}^{(m,1)} \\ S_{22,k}^{(1)} + \tilde{W}_{3,k} & \vec{J}_{n,k}^{(m,1)T} W_{1,k}^{T} \vec{J}_{n,k}^{(m,1)} \end{bmatrix} \\ 0 & \begin{bmatrix} S_{11,k}^{(2)} + \tilde{W}_{3,k} & \vec{J}_{n,k}^{(m,1)T} W_{1,k}^{T} \vec{J}_{n,k}^{(m,1)} \\ S_{22,k}^{(1)} + \tilde{W}_{3,k} & \vec{J}_{n,k}^{(m,1)T} W_{1,k}^{T} \vec{J}_{n,k}^{(m,1)} \end{bmatrix} \end{bmatrix} &
\begin{bmatrix} \begin{bmatrix} \tilde{S}_{11,k}^{(1)} \\ \hat{W}_{3,k} \end{bmatrix} \\ 0 \end{bmatrix} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\begin{bmatrix} \tilde{\Sigma}_{(10)(10)} & \bar{R}_{3,k}^{T} \\ * & \tilde{\Sigma}_{(11)(11)} \end{bmatrix} & 0 & 0 & 0 \\
* & \begin{bmatrix} \tilde{\Sigma}_{(12)(12)} & -\vec{J}_{n,k}^{(m,1)T} Q_{22,k}^{(2)} \vec{J}_{n,k}^{(m,1)} \\ * & \tilde{\Sigma}_{(13)(13)} \end{bmatrix} & 0 & 0 \\
* & * & \begin{bmatrix} \tilde{\Sigma}_{(14)(14)} & \bar{R}_{3,k}^{T} \\ * & \tilde{\Sigma}_{(15)(15)} \end{bmatrix} & 0 \\
* & * & * & \tilde{\Sigma}_{(16)(16)}
\end{array}\right] < 0 \qquad (37b)
$$

where

$$\bar{R}_{i,k} = \vec{J}_{n,k}^{(m,1)^T} \left(diag\ (\bar{R}_i\,,\ \underbrace{0_n, \cdots, 0_n}_{(k^m-m-1)\,elements}\ ) \right) \vec{J}_{n,k}^{(m,1)} \quad (i = 1, \cdots, 3)$$

$$L_k = \vec{J}_{z,k}^{(m,1)^T} \left(diag\ (\begin{bmatrix} L_0 & L_1 & \cdots & L_m \end{bmatrix},\ \underbrace{0_{z\times n}, \cdots, 0_{z\times n}}_{(k^m-m-1)\,elements}\ ) \right) \vec{J}_{n,k}^{(m,1)}$$

$$\Psi_{11} = \vec{J}_{n,k}^{(m,1)^T} Q_{11,k}^{(2)} \vec{J}_{n,k}^{(m,1)} - \sum_{i=1}^{m} \vec{J}_{n,k}^{(m,i)^T} T_{i,k}^{(1)} \vec{J}_{n,k}^{(m,i)} + \sum_{i=1}^{m} \bar{J}_{n,k}^{(m,i)^T} T_{i,k}^{(1)} \bar{J}_{n,k}^{(m,i)}$$

$$\Psi_{22} = \vec{J}_{n,k}^{(m,1)^T} Q_{22,k}^{(2)} \vec{J}_{n,k}^{(m,1)} - \sum_{i=1}^{m} \vec{J}_{n,k}^{(m,i)^T} T_{i,k}^{(2)} \vec{J}_{n,k}^{(m,i)} + \sum_{i=1}^{m} \bar{J}_{n,k}^{(m,i)^T} T_{i,k}^{(2)} \bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{11} = \vec{J}_{n,k}^{(m,1)^T} (X_{11,k}^{(3)} + X_{11,k}^{(3)^T} + \hat{Q}_{11,k}^{(1)}) \vec{J}_{n,k}^{(m,1)} - \sum_{j=1}^{m} \nu_j X_{11,k}^{\prime(1)j} + \sum_{i=1}^{m} \vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(1)} \vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m} \bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(1)} \bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{12} = \vec{J}_{n,k}^{(m,1)^T} (2\, X_{22,k}^{(1)} + \hat{Q}_{12,k}^{(1)}) \vec{J}_{n,k}^{(m,1)}$$

$$\tilde{\Sigma}_{22} = \vec{J}_{n,k}^{(m,1)^T} (X_{22,k}^{(1)} + X_{22,k}^{(1)^T} + \hat{Q}_{22,k}^{(1)}) \vec{J}_{n,k}^{(m,1)} - \sum_{j=1}^{m} \nu_j X_{22,k}^{\prime(1)j} + \sum_{i=1}^{m} \vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(2)} \vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m} \bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(2)} \bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{14} = \vec{J}_{n,k}^{(m,1)^T} W_{4,k}^{T} \vec{J}_{n,k}^{(m,1)} + \alpha \tilde{W}_{3,k}^{T}$$

$$\tilde{\Sigma}_{23} = \bar{W}_{3,k} + \tilde{W}_{3,k}$$

$$\tilde{\Sigma}_{24} = \vec{J}_{n,k}^{(m,1)^T} (W_k + (1+\alpha) W_{1,k}) \vec{J}_{n,k}^{(m,1)}$$

$$\tilde{\Sigma}_{33} = -\vec{J}_{n,k}^{(m,1)^T}(X_{11,k}^{(2)} + X_{11,k}^{(2)^T})\vec{J}_{n,k}^{(m,1)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(3)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(3)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{34} = -2\vec{J}_{n,k}^{(m,1)^T} X_{22,k}^{(1)}\vec{J}_{n,k}^{(m,1)}$$

$$\tilde{\Sigma}_{44} = -\vec{J}_{n,k}^{(m,1)^T}(X_{22,k}^{(1)} + X_{22,k}^{(1)^T})\vec{J}_{n,k}^{(m,1)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(4)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(4)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{55} = -\hat{Q}_{11,k}^{(1)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(5)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(5)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{66} = -\hat{Q}_{22,k}^{(1)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(6)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(6)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{77} = -Q_{11,k}^{(2)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(7)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(7)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{88} = -Q_{22,k}^{(2)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(8)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(8)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{99} = -\gamma^2\vec{J}_{z,k}^{(m,1)^T} diag\,(I_z,\ \underbrace{0_z,\cdots,0_z}_{(k^m-1)\,elements})\vec{J}_{z,k}^{(m,1)} + \sum_{i=1}^{m}\vec{J}_{z,k}^{(m,i)^T} Z_{i,k}^{(9)}\vec{J}_{z,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{z,k}^{(m,i)^T} Z_{i,k}^{(9)}\bar{J}_{z,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(10)(10)} = -h\bar{R}_{1,k} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(10)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(10)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(11)(11)} = -h\bar{R}_{2,k} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(11)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(11)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(12)(12)} = -Q_{11,k}^{(2)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(12)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(12)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(13)(13)} = -Q_{22,k}^{(2)} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(13)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(13)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(14)(14)} = -h\bar{R}_{1,k} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(14)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(14)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(15)(15)} = -h\bar{R}_{2,k} + \sum_{i=1}^{m}\vec{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(15)}\vec{J}_{n,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{n,k}^{(m,i)^T} Z_{i,k}^{(15)}\bar{J}_{n,k}^{(m,i)}$$

$$\tilde{\Sigma}_{(16)(16)} = -\vec{J}_{s,k}^{(m,1)^T} diag\,(I_s,\ \underbrace{0_s,\cdots,0_s}_{(k^m-1)\,elements})\vec{J}_{s,k}^{(m,1)} + \sum_{i=1}^{m}\vec{J}_{s,k}^{(m,i)^T} Z_{i,k}^{(16)}\vec{J}_{s,k}^{(m,i)} - \sum_{i=1}^{m}\bar{J}_{s,k}^{(m,i)^T} Z_{i,k}^{(16)}\bar{J}_{s,k}^{(m,i)}$$

then the state-space matrices of the observer-based H_∞ control are given by

$$F_\rho = \left((\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T X_{22,k}^{(1)}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)\right)^{-1}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T W_k^T(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n), \tag{38a}$$

$$F_{1\rho} = \left((\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T X_{22,k}^{(1)}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)\right)^{-1}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T W_{1,k}^T(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n), \tag{38b}$$

$$F_{2\rho} = \left((\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T Q_{22,k}^{(2)}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)\right)^{-1}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T W_{2,k}^T(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n), \tag{38c}$$

$$G_\rho = \left((\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T X_{22,k}^{(1)}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)\right)^{-1}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T W_{3,k}^T(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n), \tag{38d}$$

$$K_\rho = (B_\rho^T B_\rho)^{-1}B_\rho^T\left((\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T X_{11,k}^{(1)}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)\right)^{-1}(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n)^T W_{4,k}^T(\otimes_{i=m}^{1}\rho_i^{[k]}\otimes I_n), \tag{38e}$$

$$K_{1\rho} = (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_z)^T K_{1,k} (\otimes_{i=m}^{1} \rho_i^{[k]} \otimes I_n). \quad (38f)$$

Proof: By substituting the relations (28)–(36) into the PLMI (20b), one parameter-dependent matrix inequality is obtained which includes left-and right-multiplication of the LMI (20b) by:

$$\begin{bmatrix} \overbrace{\begin{matrix} \left(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n\right)^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ * & \cdots & \left(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n\right)^T \end{matrix}}^{8\,times} & 0 & 0 & 0 \\ * & \left(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_z\right)^T & 0 & 0 \\ * & * & \overbrace{\begin{matrix} \left(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n\right)^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ * & \cdots & \left(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_n\right)^T \end{matrix}}^{3\,times} & 0 \\ * & * & * & \left(\otimes_{i=m}^{1} \rho_i^{[k+1]} \otimes I_s\right)^T \end{bmatrix}$$

and its transpose. Then, it can be concluded that LMI (37b), which included the positive definite Lagrange multiplier matrices $\{Z_{i,k}^{(j)}\}_{i=1,\cdots,m}^{j=1,\cdots,16}$ (see Bliman, 2004b; 2005) to obtain less conservative results, are sufficient conditions to fulfill the matrix inequality (20b) for any vector-valued parameter $\rho \in \zeta$ in Assumption 1. Similar to the procedure above with the positive definite Lagrange multiplier matrices $\{T_{i,k}^{(1)}, T_{i,k}^{(2)}\}_{i=1,\cdots,m}$, the LMI (37a) is concluded.

Remark 7: We note that the problem of finding the smallest $\gamma > 0$, namely γ_0, is to determine whether the problem (37) is feasible or not. It is called the feasibility problem. Also, the solutions of the problem can be found by solving generalised eigenvalue problem in

$X_{11,k}^{(1)}, X_{22,k}^{(1)}, Q_{11,k}^{(2)}, Q_{22,k}^{(2)}, \hat{Q}_{11,k}^{(1)}$,

$\hat{Q}_{22,k}^{(1)}, \overline{R}_1, \overline{R}_2$,

$W_k, W_{1,k}, W_{2,k}, W_{3,k}, W_{4,k}, K_{1,k}, X_{11,k}^{(2)}, X_{11,k}^{(3)}, \hat{Q}_{12,k}^{(1)}, \overline{R}_3$,

$\{Z_{i,k}^{(j)}\}_{i=1,\cdots,m}^{j=1,\cdots,16}$, $\{T_{i,k}^{(1)}, T_{i,k}^{(2)}\}_{i=1,\cdots,m}$, α

and

$\lambda = \gamma^2$,

which is a quasi-convex optimisation problem as follows:

$$\begin{gathered} \textit{Minimize} \quad \lambda \\ \textit{subject to} \;\; X_{11,k}^{(1)} > 0, X_{22,k}^{(1)} > 0, Q_{11,k}^{(2)} > 0, Q_{22,k}^{(2)} > 0, \hat{Q}_{11,k}^{(1)} > 0, \hat{Q}_{22,k}^{(1)} > 0, \overline{R}_1 > 0, \overline{R}_2 > 0, Z_{i,k}^{(j)} > 0, \\ T_{i,k}^{(1)} > 0, T_{i,k}^{(2)} > 0, W_k, W_{1,k}, W_{2,k}, W_{3,k}, W_{4,k}, K_{1,k}, X_{11,k}^{(2)}, X_{11,k}^{(3)}, \hat{Q}_{12,k}^{(1)}, \overline{R}_3, \alpha, \textit{and LMIs } (37) \end{gathered}$$

such the bound γ_0 is given by $\gamma_0 = \sqrt{\lambda^*}$, where λ^* is the optimal value of the optimisation problem. Note that a locally optimal point of a quasi-convex optimisation problem with strictly quasi-convex objective is globally optimal (for details, see Boyd et al., 1994). Various efficient convex optimisation algorithms can be used to check whether the matrix inequalities (37) are feasible. In this paper, in order to solve the matrix inequalities (37), we utilise Matlab's LMI Control Toolbox (Gahinet et al., 1995), which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimisation algorithms (Boyd et al., 1994). Of course, the high dimension of the resulting LMI will increase the computational complexity of the proposed approach to some extent. It turns out that, by working with symmetry reduction techniques (Gaterman and Parillo, 2004), a smaller size of LMI can be derived with a significantly smaller number of decision variables so that the computational burden can be reduced. This notably affects the total running time. Therefore, the reduction techniques enable the numerical solution of large scale instances that are otherwise computationally infeasible to be solved.

Remark 8: A new set of matrices verifying LMIs (37) can be generated, with index $k+1$ instead of k. In this case, the solvability of LMIs (37) implies the same property for the larger values of the index k. Of course, the high dimension of the resulting LMIs will increase the computational complexity of the proposed approach to some extent. However, our approach is different from those in Mahmoud and Boujarwah (2001); Velni and Grigoriadis (2007); Wu et al. (2006) in several perspectives:

a their system structure corresponds to our case with $A_2(\rho) = 0$, i.e. an LPV retarded system and $B(\rho) = 0$ where other state-space matrices are real continuous matrix functions which affinely depend on the vector-valued parameter ρ

b the augments states are defined differently

c their test conditions are a set of PLMIs conditions while ours are PLMIs with relaxation embedded

d this paper presents a systematic approach for the delay-dependent observer-based H_∞ control of LPV neutral systems using PPDQ function that is never seen before.

It is also worth mentioning that, effective use of our results is subordinate to powerful LMI solvers. According to Bliman (2005), a general idea for reducing the computation complexity (or computational burden) consists in performing first a subdivision of the admissible parameter set in sub-domains and applying the results given in Theorem 3.

4 Illustrative example

Consider the following state-space matrices for the LPV neutral system (1),

$$A_0 = \begin{bmatrix} -1 & 0.4 \\ 0.05 & -0.5 \end{bmatrix};$$

$$A_1 = A_{11} = A_{21} = diag\{-0.2, -0.1\};$$

$$A_{10} = \begin{bmatrix} -0.5 & 0.1 \\ 0.4 & -1 \end{bmatrix};\ A_{20} = diag\{-0.3, -0.6\};$$

$$E_{10} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};\ B_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}; E_{20} = 1;\ C_0 = L_0 = [1 \quad 1]$$

where $\rho(t) = 0.5\sin(t)$ and the constant delays are assumed to be $h = \tau = 0.5$. For simulation purpose, a unit step signal in the time interval $[0, 1]$ as the disturbance is imposed on the system.

It is required to design an observer-based H_∞ control such that the closed-loop system is asymptotically stable and satisfies the H_∞ performance measure. To this end, in light of Theorem 3 with $k = 2$ and the performance bound $\gamma = 0.95$, we solved LMIs (37) and obtained the control and the observer gains in (3) for the whole range of the parameter $\rho(t)$, for instance

$$F(\rho)|_{\rho=0.5} = \begin{bmatrix} -0.852 & 0.1852 \\ 0.1288 & -0.244 \end{bmatrix},$$

$$F_1(\rho)|_{\rho=0.5} = \begin{bmatrix} 0.3460 & -0.5386 \\ -0.4083 & 0.8260 \end{bmatrix},$$

$$F_2(\rho)|_{\rho=0.5} = \begin{bmatrix} -0.6140 & -0.1969 \\ 0.2120 & -0.1947 \end{bmatrix},$$

$$G(\rho)|_{\rho=0.5} = \begin{bmatrix} -0.0886 \\ -0.0761 \end{bmatrix},$$

$$K_1(\rho)|_{\rho=0.5} = [-0.8422 \quad -0.6467],$$

$$K(\rho)|_{\rho=0.5} = [3.5616 \quad 2.2551].$$

For initial condition $(x_1(0), x_2(0)) = (1, 0)$, the simulation results are shown in Figures 1–3. The curve of observer-based H_∞ control in (3d) is shown in Figure 1. To observe the H_∞ performance, the response of the estimation error signal, i.e., $e(t) = z(t) - \hat{z}(t)$, is depicted in Figure 2 and correctness of the disturbance attenuation level, i.e. $\|e(t)\|_2^2 - \gamma^2 \|w(t)\|_2^2 < 0$, is plotted in Figure 3. It is seen from Figures 2 and 3 that the closed-loop system is asymptotically stable and the control signal reduces the effect of the disturbance input $w(t)$ on the estimation error. The solution was obtained after about 80 seconds on a computer with a 2.67 GHz Pentium processor.

Figure 1 Control law for system (see online version for colours)

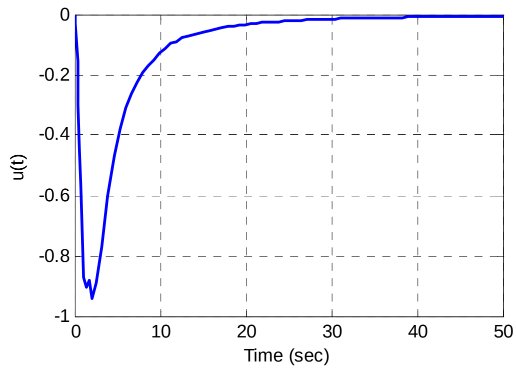


Figure 2 Curve of estimation error signal (see online version for colours)

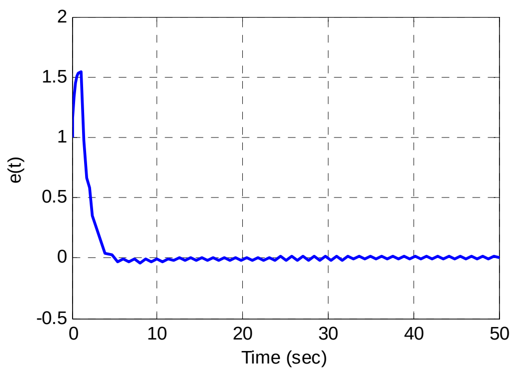
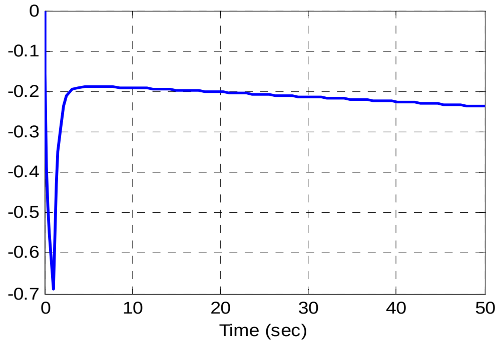


Figure 3 Curve of $\|e(t)\|_2^2 - \gamma^2 \|w(t)\|_2^2$ (see online version for colours)



5 Conclusions

The convex optimisation problem of robust observer-based control for a class of LPV neutral systems with constant time-delays has been studied in this paper. A delay-dependent observer-based H_∞ control, which depends on parameters that are measurable in real time, has been proposed. By using the polynomial parameter-dependent quadratic (PPDQ) functions and a suitable change of variables, the required sufficient conditions with high precision have been established in terms of delay-dependent parameter-independent LMIs for the existence of the desired observer-based control. However, the explicit expression of the robust delay-dependent observer-based control has been derived to satisfy both asymptotic stability and H_∞ performance. A numerical example has been provided to demonstrate the usefulness of the theory developed.

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