## AUTONOMOUS CONTROL OF SHOP FLOOR LOGISTICS: ANALYTIC MODELS

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Abstract: In complicated shop floor environments of hundreds of machines it is difficult to organize an effective and robust central control strategy for all processing parts, buffers and machines. We consider a production line for several types of products produced in parallel lines of machines. For this problem we derive a continuous model using ordinary differential equations and discuss the corresponding optimal control problem. Here optimality criteria concern the idle time of machines, work-in-process as well as the total throughput of the production. To develop a robust and close to optimal control mechanism we study the case where the individual parts waiting for a processing step have the capability to choose a machine themselves. We discuss different strategies for this decision, depending on the desired optimization objective, and their advantages in comparison to simple parallel production and globally optimized solutions. These strategies may be interpreted as local autonomous control. The main advantages of the approach lie in the comparatively simple implementation of the control mechanism as well as in the added robustness, when compared to a pre-planned production schedule.

Keywords: Shop-floor systems, autonomous control, optimization

## 1. INTRODUCTION

In complicated shop floors consisting of hundreds of machines it is difficult to organize an effective and robust central control strategy for all processing parts, buffers and machines. In this paper we consider some simple production scenarios to demonstrate the idea of a possible autonomous production. Similar configurations have been simulated with stochastic models in (Scholz-Reiter *et al.*, 2004), where the advantages of an autonomous control are discussed.

We are interested in the following scenario. Consider a shop floor with n production lines. Each of the lines is optimized to the processing of a specific type of parts. However, in case a certain line is idle, this line can be used to process another type of parts, albeit with reduced efficiency. Thus there are kinds of parts indexed by  $i = 1, \ldots, n$  that each arrive with a time-dependent rate  $a_i(t), i = 1, \ldots, n$ . The *i*-th kind part can be processed by the *j*-th production line with a production rate of  $b_{ij}, i, j = 1, \ldots, n$ . We assume for all  $i, j = 1, \ldots, n$  that  $b_{ii} > b_{ij}, j \neq i$ , and also that  $b_{ii} > b_{ji}$ . These two assumptions mean that the *i*-th line produces the *i*-th kind product more efficiently than any other line does and that also the *i*-th production line does not process parts of another kind faster than the *i*-th one. In principle, we can think of production lines consisting of several stages, where the output of each stage becomes the input of the next stage. As the output of the previous stage may be interpreted as the arrival of parts to be processed, we concentrate here of a one-stage scenario. In a first step we describe a continuous model for the previous setup. For this an obvious optimal control strategy can be presented. We will argue that the implementation of this approach is impractical for large shop floors. It is the aim of the present paper to discuss distributed, local control strategies, that may be interpreted as autonomous regulation.

We now present a continuous model for this situation. Continuous models, also called fluid models, see e.g., (Kleinrock, 1975), (Armbruster, 2004). They allow to describe the material flows in terms of differential equations.

The arrival of the *i*-th raw material is governed by the time dependent function  $a_i(t)$ , which we assume to be a piecewise continuous, nonnegative function. The raw materials (parts) are stored in a single central buffer. The state vector of the buffer is given by  $x(t) = (x_1(t), \ldots, x_n(t))$ , where  $x_i(t)$ denotes the amount of the *i*-th raw material stored in the buffer. The time derivatives of the *i*-th state is given by

$$\dot{x}_{i}(t) = a_{i}(t) - f_{i}(a_{i}(t), x_{i}(t)) - \sum_{i \neq j} g_{ij}(a(t), x(t)) .$$
(1)

Here the function  $f_i$  describes the reduction in the *i*-th raw material due to the production process in the *i*-th production line, whereas the functions  $g_{ij}$  denote the reduction in the stored raw material due to processing of the *i*-th part by the *j*-th line for  $i \neq j$ .

If we want to maximize the throughput, then as the *i*-th machine works in an optimal manner with the *i*-th part. Then as long as there is a supply of the *i*-th raw material it is optimal to use the *i*th production line exclusively for this. That is we define

$$f_i(a_i, x_i) = \begin{cases} b_{ii} &, \text{ if } x_i > 0\\ \min\{a_i, b_{ii}\} &, \text{ if } x_i = 0 \end{cases}$$
(2)

The terms  $g_{ij}$  are used to distribute material for the *i*-th product to the *j*-th production line in case this line still has some capacity. We assume that a, x are given and let  $i_1, \ldots, i_m$  denote indices for which  $x_{i_l} = 0, l = 1, \ldots, m$ . (If there are no such indices, then all production lines work with their corresponding product, and there is nothing to discuss.) The optimal way to do this is given by the solution of the following linear program.

Maximize 
$$\sum_{i \neq j=1}^{n} g_{ij}$$
 (3)

subject to

$$g_{ij} \ge 0$$
,  $i, j = 1, \dots, n, i \ne j$ , (4)

$$\sum_{j=1, j\neq i}^{n} g_{ij} \le a_i - f_i(a_i, x_i), \ i = i_1, \dots, i_m, \ (5)$$

$$\sum_{i=1,i\neq j}^{n} b_{ij}^{-1} g_{ij} \le \left(1 - \frac{f_j(a_j, x_j)}{b_{jj}}\right), \ j = 1, ..., n.(6)$$

The constraints represent to positivity constraint on the  $g_{ij}$ , the constraint that not more than  $a_i - f_i(a_i, x_i)$  can be processed of the *i*-th product, in case that the buffer for the *i*-th product is empty, and finally, that the total capacity of the *j*-th machine may not be exceeded.

This optimization problem needs to be solved whenever some of the buffers become empty. In this way the amount of material delivered to a machine is exactly equal to the amount that can be processed, so that there is no build up of material other than in the buffer. Also it is easy to see that at each time instant the throughput is maximized and idle times are minimized. This is the content of the following statement.

Proposition 1. Consider the production line described by the equations (1), i = 1, ..., n with a distribution according to (2) and the linear program (3)–(6). Then for all T > 0 this distribution maximizes the throughput, which is given by

$$\sum_{j=1}^{n} \int_{0}^{T} f_{j}(a_{j}(t), x_{j}(t)) + \sum_{i=1, i \neq j}^{n} g_{ij}(a(t), x(t)) dt.$$

Furthermore, the distribution procedure minimizes the total idle times of the production lines.

While the described approach yields an optimal solution, the described method has several drawbacks, so that it is not feasible in shop floors characterized by a large number of production lines. First of all, the solution of the linear program may be cumbersome and it requires full information concerning the states of all machines and the total work-in-process. Secondly, the common buffer may be impractical depending on the circumstances, or in case of a virtual buffer further communication needs may be undesirable.

In the following we will investigate decentralized control strategies. We present indications that solutions close to optimal may be obtained using autonomous control concepts. For ease of presentation we restrict ourselves to the case of two production lines. In the following Section 2 we present three different scenarios for this case. In Section 3 some discrete event simulations for these scenarios are discussed. Finally, in Section 4 we present continuous models for these situations and discuss some optimal control problems.

## 2. THE CASE OF TWO PRODUCTION LINES

Let us consider a production of two kinds of products in two lines. There are two kinds of parts to be processed coming from two sources (scenario 1, see Fig. 1) with rate  $a_1$  and  $a_2$  (parts per unit time), respectively. The processing rate of machines is denoted by  $b_{11}$  and  $b_{22}$  respectively.



Fig. 1. Scenario 1



Fig. 2. Scenario 2

In the case that  $a_i > b_{ii}$ , i = 1, 2 the queues in the buffers grow linearly with time. If  $a_i < b_{ii}$ , the machines idle periodically. Now let  $a_1 < b_{11}$  and  $a_2 > b_{22}$  ( $a_1 > b_{11}$  and  $a_2 < b_{22}$  is symmetric). In this case the first machine idles periodically whereas the second has to proceed a growing queue.

To save the idle time of the machine 1 one can allow parts to choose another machine if this machine idles (scenario 2, Fig. 2). Let us denote  $b_{ij}$  the processing rate of the machine *i* busy with part from the source *j*.

We consider also the following scenario 3. The parts arrive first to a common buffer and then decide to which machine to go (Fig. 3), whereby a part has a preference to go to the machine with a highest processing rate for it.

To demonstrate the advantages of the last two scenarios we perform a discrete event simulation.



#### Fig. 3. Scenario 3

### 3. DISCRETE EVENT SIMULATION

We normalize the maximal arrival rate of source 2 to  $a_2 = 1$ . It is clear, that the interesting case is  $a_1 < b_{11}$  and  $a_2 > b_{22}$ . The critical case occurs when the processing times are significantly smaller then possible arrival rates of one of the servers. Thus we set  $a_1 = 1/24, b_{11} = b_{22} = 1/16, b_{12} = b_{21} = 1/20$  and vary  $1/16 < a_2 < 1$ , as smaller values of  $a_2$  lead to scenarios similar to the previous one. The simulation time period is 500 units. The parts of the kind *i* go to the machine  $j \neq i$  if and only if it idles.

On following figures dash, solid and dotted lines correspond to scenario 1, 2 and 3 respectively. The total amount of parts processed by both machines is presented on the Fig. 4.



Fig. 4. Total throughput versus inter-arrival time  $1/a_2$ 

As one can expect the three lines coincide if interarrival time  $\frac{1}{a_2} > 16$ , i.e., for low arrival rates. In this case all three scenarios work in the same parallel way. For higher arrival rates we see that with the exception of two points  $a = \frac{1}{8}$  and  $a = \frac{1}{12}$  the throughput in the second scenario is bigger as in the first one. The third scenario has a bigger throughput than in scenarios 1 and 2 or all  $\frac{1}{a_2} < 16$  as we expect from Proposition 1.

The sum of the idle time over both machines is presented on the Fig. 5.



Fig. 5. Total idle time versus inter-arrival time  $1/a_2$ 



Fig. 6. Mean work-in-process versus inter-arrival time  $1/a_2$ 

Again we see, that with the exception of the same two points the second scenario has less total idle time and the third scenario has less idle time than both scenarios 1 and 2 for all  $\frac{1}{a_2} < 16$ .

Figures 6 and 7 show the mean and the maximum work-in-process, i.e. the amount of parts in both buffers in the first two scenarios.



Fig. 7. Maximum work-in-process versus interarrival time  $1/a_2$ 

We see that nearly the same buffer capacities are required in all three cases. However, the last two production scenarios have advantages in that the idle time is less and the total throughput is bigger.

#### 4. CONTINUOUS MODELS

#### 4.1 Scenario 1

As before let  $a_i(t)$  be the time dependent rate of arrivals from the source *i* and  $b_{ii}$  be the processing rate of the machine *i*. Let  $x_i(t), i = 1, 2$  be the amount of parts waiting in the first and second buffer. As there is no cross transfer of raw material (1) reduces to

$$\dot{x}_i(t) = a_i(t) - f_i(a_i(t), x_i(t)), \quad i = 1, 2, (7)$$

with processing rates  $f_i$  given by (2).

The total throughput and idle time as well as the queue length of the buffers can be calculated easily. There are no controls in this scenario. It is clear, that the non-interaction between the different production lines may lead to solutions that are far from optimal. In particular, idle times occur whenever a buffer becomes empty and these idle times cannot be used for other purposes, so that also the throughput is reduced. We do not discuss this simple case further.

### 4.2 Scenario 2

Let  $a_i(t)$  and  $b_{ij}$  be as above. Let  $x_1(t), y_1(t)$  be the number of parts in the buffer 1 coming from the source 1 and 2 respectively and let  $x_2(t), y_2(t)$ be the number of parts in the buffer 2 coming from source 2 and 1 respectively. Let  $0 \le \alpha_i(t) \le 1$ be two time dependent parameters controlling the rate of parts arrived from the source *i* and going to the machine 1. Then the evolution of the buffer queues can be described by the following system:

$$\dot{x}_1(t) = \alpha_1 a_1(t) - b_1(t) \frac{x_1(t)}{x_1(t) + y_1(t)},$$
 (8)

$$\dot{y}_1(t) = \alpha_2 a_2(t) - b_1(t) \frac{y_1(t)}{x_1(t) + y_1(t)},\tag{9}$$

$$\dot{x}_2(t) = (1 - \alpha_2)a_2(t) - b_2(t)\frac{x_2(t)}{x_2(t) + y_2(t)} (10)$$

$$\dot{y}_2(t) = (1 - \alpha_1)a_1(t) - b_2(t)\frac{y_2(t)}{x_2(t) + y_2(t)}$$
(11)

where  $b_1(t)$  and  $b_2(t)$  are the production rates of the machine 1 and 2 respectively. They can be calculated as follows: Consider the first machine and the queue  $(x_1 + y_1)$  in the first buffer. If  $(x_1 + y_1) > 0$ , then let  $\varepsilon(x_1 + y_1)$  be a small portion of the queue processed by this machine. The time spent for this portion is  $\left(\frac{\varepsilon x}{b_{11}} + \frac{\varepsilon y}{b_{12}}\right)$ . Then it follows

$$b_1(t) = \frac{\varepsilon(x_1(t) + y_1(t))}{\frac{\varepsilon x_1(t)}{b_{11}} + \frac{\varepsilon y_1(t)}{b_{12}}}.$$
 (12)

With the same arguments for the second machine one can conclude:

$$b_i(t) = \frac{x_i(t) + y_i(t)}{\frac{x_i(t)}{b_{ii}(t)} + \frac{y_i(t)}{b_{ij}(t)}}, \quad i \neq j.$$
(13)

For  $x_1 + y_1 = 0$ , i.e.  $x_1(t) = y_1(t) = 0$  we conclude in the same way that

$$b_1 = \frac{\alpha_1 a_1 + \alpha_2 a_2}{\frac{\alpha_1 a_1}{b_{11}} + \frac{\alpha_2 a_2}{b_{12}}}$$

if the machine does not idle and  $b_1 = \alpha_1 a_1 + \alpha_2 a_2$  otherwise (i.e., for small arrival rates). So we obtain

$$b_1(t) = \min\left(\alpha_1 a_1 + \alpha_2 a_2, \frac{\alpha_1 a_1 + \alpha_2 a_2}{\frac{\alpha_1 a_1}{b_{11}} + \frac{\alpha_2 a_2}{b_{12}}}\right), \quad (14)$$

for 
$$x_1(t) = y_1(t) = 0;$$
  
 $b_2(t) = \min\left((1 - \alpha_1)a_1 + (1 - \alpha_2)a_2, \frac{(1 - \alpha_1)a_1 + (1 - \alpha_2)a_2}{\frac{(1 - \alpha_1)a_1}{b_{12}} + \frac{(1 - \alpha_1a_2)}{b_{22}}}\right),$  (16)  
for  $x_2(t) = y_2(t) = 0$ 

for  $x_2(t) = y_2(t) = 0.$ 

The control parameters  $\alpha_i$  should be chosen to reach an optimal solution.

The criteria for the optimal solution can be for example:

• maximizing the total throughput over a time interval [0, T]

$$\int_{0}^{T} (b_1(s) + b_2(s)) ds \to \max; \qquad (17)$$

• minimizing the total work-in-process

$$x_1(t) + y_1(t) + x_2(t) + y_2(t) \rightarrow \min;$$
 (18)

• minimizing the total idle time of the first machine:

$$T - \frac{\int_{0}^{T} \alpha_{1}(t)a_{1}(t) dt}{b_{11}} - \frac{\int_{0}^{T} \alpha_{2}(t)a_{2}(t) dt}{b_{21}} \to \min;$$
(19)

Then an optimal control problem (Macki and Strauss, 1982), (Fleming and Rishel, 1975) can be formulated as follows:

Find  $\alpha_1, \alpha_2$  such that the solution of (8-16) yields an optimal return for the functional (17), (18) or (19), respectively.

We note, that  $b_i(t)$  is discontinuous at  $x_i(t) = y_i(t) = 0$ , hence the uniqueness of the solution of the system (8-11) is not clear at that point. To investigate this case we do the following transformation:

$$u = x_1 + y_1$$
  

$$v = x_1 - y_1$$
  

$$\xi = x_2 + y_2$$
  

$$\eta = x_2 - y_2$$

$$\begin{aligned} \dot{u} &= \alpha_1 a_1 + \alpha_2 a_2 - b_1 \\ \dot{v} &= \alpha_1 a_1 - \alpha_2 a_2 - b_1 \frac{v}{u} \\ \dot{\xi} &= (1 - \alpha_2) a_2 + (1 - \alpha_1) a_1 - b_1 \\ \dot{\eta} &= (1 - \alpha_2) a_2 - (1 - \alpha_1) a_1 - b_2 \frac{\eta}{\xi} \end{aligned}$$

where now  $b_1$  may be discontinuous only if u = v = 0 and  $b_2$  may be discontinuous only if  $\xi = \eta = 0$ . The initial conditions now are u(0) = 0, v(0) = 0,  $\xi(0) = 0$ ,  $\eta(0) = 0$ .

Let  $(u_1, v_1, \xi_1, \eta_1)$  and  $(u_2, v_2, \xi_2, \eta_2)$  be two solutions of this system, then

$$(u_1 - u_2)^{\cdot} = 0 \tag{20}$$

$$(v_1 - v_2)^{\cdot} = b_1 \left(\frac{v_1}{u_1} - \frac{v_2}{u_2}\right) \tag{21}$$

$$(\xi_1 - \xi_2)^{\cdot} = 0 \tag{22}$$

$$(\eta_1 - \eta_2)^{\cdot} = b_2 \left(\frac{\eta_1}{\xi_1} - \frac{\eta_2}{\xi_2}\right)$$
 (23)

with  $(u_1 - u_2)(0) = 0$ ,  $(v_1 - v_2)(0) = 0$ ,  $(\xi_1 - \xi_2)(0) = 0$ ,  $(\eta_1 - \eta_2)(0) = 0$ . Firstly, it follows that  $u_1 - u_2 \equiv 0$ ,  $v_1 - v_2 \equiv 0$ , then

$$\xi_1 - \xi_2 = C_1 \exp(\int \frac{b_1}{u_1} dt), \qquad (24)$$

$$\eta_1 - \eta_2 = C_2 \exp(\int \frac{b_2}{\xi_2} dt).$$
 (25)

With homogeneous initial conditions it follows  $\xi_1 - \xi_2 \equiv 0, \ \eta_1 - \eta_2 \equiv 0$ . The uniqueness is proved. In order to maximize the throughput we now describe how to find  $\alpha_i(t)$  that instanteneously

maximize the production rate, i.e.  $b_1(t) + b_2(t)$ . We consider the system in different buffer states: Obviously,  $\alpha_1(t) = 1$  and  $\alpha_2(t) = 0$  for  $x_1(t) + y_1(t) > 0$  and  $x_2(t) + y_2(t) > 0$ , i.e., the machine *i* receives the parts only from the source *i*, *i* = 1, 2.

Let be  $x_1(t) + y_1(t) = 0$  and  $x_2(t) + y_2(t) > 0$ . Then  $\alpha_1(t) = 1$ , since the second machine is busy. If  $a_1 \ge b_{11}$ , i.e., arrival rate is higher then it can be processed, then it follows  $\alpha_2(t) = 0$ . Otherwise if  $a_1 < b_{11}$ , the first machine can process some parts from the second source and having empty buffer. To find the appropriate  $\alpha_2(t)$  we use the condition of empty buffer: let us consider a small time interval  $\Delta t$ . During that time  $a_1\Delta t$  parts have arrived from the first source. The first machine has spent  $\frac{a_1\Delta t}{b_{11}}$  time units processing them. The remaining time  $\Delta t - \frac{a_1\Delta t}{b_{11}}$  can be used for the parts of the second machine which are processed with the rate  $b_{12}$ . It follows that

$$\left(\Delta t - \frac{a_1 \Delta t}{b_{11}}\right) b_{12} = \alpha_2 a_2,\tag{26}$$

if  $a_2(t) > 0$  is big enough, such that there is no idle time. Finally

$$\alpha_2(t) = \min\Big(\frac{(b_{11}(t) - a_1(t))_+ b_{12}(t)}{b_{11}(t)a_2(t)}, 1\Big),$$

where we use the notation  $a_{+} = \max(0, a)$ .

The case  $x_1(t) + y_1(t) > 0$  and  $x_2(t) + y_2(t) = 0$ can be treated similarly to obtain

$$\alpha_1(t) = \max\left(0, 1 - \frac{(b_{22}(t) - a_2(t)) + b_{21}(t)}{b_{22}(t)a_1(t)}\right),$$
  
and  $\alpha_2(t) = 0.$ 

The last possible state is that both buffers are empty, i.e.  $x_1(t) + y_1(t) = 0$  and  $x_2(t) + y_2(t) = 0$ . Straightforward calculations yield in this case

$$\alpha_1(t) = \min\left(1, \left(1 - \frac{(b_{22}(t) - a_2(t))_+ b_{21}(t)}{b_{22}(t)a_{11}(t)}\right)_+\right),$$
  
$$\alpha_2(t) = \max\left(0, \frac{(b_{11}(t) - a_1(t))_+ b_{12}(t)}{b_{11}(t)a_2(t)}\right).$$

For this  $\alpha_1(t), \alpha_2(t)$  the solution  $x_1(t), y_1(t), x_2(t), y_2(t)$  can be found solving the system (8-11).

# 4.3 Scenario 3

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Now consider the third scenario. Again let  $a_i(t)$ ,  $b_{ij}(t)$  be as above. Let x(t) and y(t) denote the number of parts waiting in the buffer arrived from the source 1 and 2 respectively. Let  $0 \leq \alpha_i \leq b_{1i}$ ,  $0 \leq \beta_i \leq b_{2i}$  denote the rate of arrival of parts coming from the source 1 and 2 respectively and going to the machine *i*. The evolution of x(t) and y(t) is then given by:

$$\dot{x} = a_1(t) - \alpha_1(t) - \alpha_2(t),$$
 (29)

$$\dot{y} = a_2(t) - \beta_1(t) - \beta_2(t).$$
 (30)

Since the parts go to a machine from the buffer only if it becomes empty, i.e., the arrival rate is equal to the processing rate. Then

$$\alpha_i(t) + \beta_i(t) = b_i(t), \quad i = 1, 2$$
 (31)

where  $b_1(t)$  and  $b_2(t)$  are the processing rates of the machine 1 and 2 respectively. Using (31) the system (29-31) is equivalent to

$$\dot{x}(t) = a_1(t) - \alpha_1(t) - (b_2(t) - \beta_2(t))$$
 (32)

$$y(t) = a_2(t) - (b_1(t) - \alpha_1(t)) - \beta_2(t)$$
(33)

$$b_1^2(t) = \alpha_1(t)b_{11}(t) + (b_1(t) - \alpha_1(t))b_{12}(t) (34)$$

$$b_2^2(t) = (b_2(t) - \beta_2(t))b_{21}(t) + \beta_2(t)b_{22}(t)$$
(35)

and one has only two independent control functions  $\alpha_1(t)$  and  $\beta_2(t)$ .

Let us find  $\alpha_1(t), \alpha_2(t), \beta_1(t), \beta_2(t)$  minimizing the work-in-process (x + y): First we treat the case of empty buffers x = 0, y = 0. Analyzing the Fig. 8 and with the same arguments as in case of scenario 2 one the following solution is obtained

$$\alpha_1 = \min(b_{11}, a_1),$$



Fig. 8. Area of arrival rates variation

$$\alpha_{2} = \min\left(\left(1 - \frac{a_{2}}{b_{22}}\right)_{+} b_{21}, a_{1} - b_{11}\right),$$
  
$$\beta_{1} = \min\left(\left(1 - \frac{a_{1}}{b_{11}}\right)_{+} b_{12}, a_{2} - b_{22}\right),$$
  
$$\beta_{2} = \min(b_{22}, a_{2}),$$

where we use the notation  $a_{+} = \max(0, a)$ .

In case x > 0, y > 0 it follows that  $\alpha_1 = b_{11}$ ,  $\alpha_2 = 0$ ,  $\beta_1 = 0$ ,  $\beta_2 = b_{22}$ .

For x > 0, y = 0 one has  $\alpha_1 = b_{11}, \beta_1 = 0, \beta_2 = \min(b_{22}, a_2)$  and

$$\alpha_2 = \min\left(\left(1 - \frac{a_2}{b_{22}}\right)_+ b_{21}, a_1 - b_{11}\right),\$$

and for x = 0, y > 0 we have  $\alpha_1 = \min(b_{11}, a_1), \beta_1 = 0, \beta_2 = b_{22}$  and

$$\alpha_2 = \min\left(\left(1 - \frac{a_2}{b_{22}}\right)_+ b_{21}, a_1 - b_{11}\right).$$

We have found optimal controls  $\alpha_i$  for all cases, with these data the evolution of the buffer queue x(t), y(t) is given by (29-31).

### 5. CONCLUSION

Several scenarios of shop floor control have been considered and it has been demonstrated that scenarios with autonomous control strategies, in which parts can decide locally which machine to go to, can be more effective than conventional preplanned schedules where parts are handed over to the next machine in line according to a production plan.

The scenarios have been described by means of analytical models in form of differential equations and control functions in order to analyze the performance of distributed autonomous control systems. Several criteria of optimality, such as minimum idle times of machines, minimum work-in-process and maximum throughput have been stated. The optimal control functions can be found by solving the corresponding optimal control problem. Moreover it has been shown that autonomous shop floor control is more robust in the case of machine breakdowns. Here, simple rules of local decision-making allow jobs to be transferred to another machine. These advantages have also been confirmed by simulations described in (Scholz-Reiter *et al.*, 2004) using a different modeling approach.

The presented research on decentralized and autonomous control scenarios has established a theoretical foundation for further research. The results have proven that it is promising to focus on more complex shop floor structures. In such a scenario the autonomy of a single part is more important as far as its influence on the performance in terms of less idle times of machines, higher throughput and less total work-in-process as well as the system's robustness is concerned.

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